

Significant Figures

Why Worry About These?

The number of digits, i.e., significant figures, reported for a numerical quantity conveys the quality of a measurement or analysis to the reader. In any work involving numerical values, the precision of these values, which is represented by the number of digits shown, is important information. In general chemistry, you will have to use a meaningful number of digits in reporting your results.

Examples: Accuracy, Precision, and Significant Figures

- *Accuracy* is the degree of conformity to a standard or true value.
- *Precision* is the smallest repeatable digit of a measurement.
- *Significant figures* are the repeatable digits and the first uncertain digit in a measurement or calculation.

Item	Values	Significant Figures
Bureau of Standards time	9:15:13004	?
Jerry's analog watch	9:15	3
Jennifer's digital watch	9:17:52	5
average mass of a coin	23.32 g	4
Height of an index card	0.0770 m	3

Questions About the Examples

1. The Bureau of Standards time is, by definition, the true value of time. To how many significant figures is this time known? *8 digits*

2. Whose watch is more accurate, Jerry's or Jennifer's? Explain.

Its time is closer to the true time

3. Whose watch is more precise? Explain.

Jennifer's - its time is known to 5 s.f. Jerry's watch displays time to only 3 s.f.

4. How do we represent precision in reporting a measurement?

Through the number of digits reported.

5. Why is the height of the index card reported as 0.0770m and not 0.077m?

It must have been measured to the nearest 0.0001m (0.1mm).

Examples: Arithmetic Operations and Significant Figures

Addition

$$\begin{array}{r}
 23.26 \text{ g} \\
 \text{least precisely known number} \rightarrow 100.1 \text{ g} \\
 0.03631 \text{ g} \\
 \hline
 \end{array}$$

123.39631 g
significant → ← not significant

1st uncertain digit
report as 123.4 g

all digits are 'significant'

Subtraction

$$\begin{array}{r}
 45.8 \text{ g} \\
 - 3.26 \text{ g} \\
 \hline
 \end{array}$$

42.54 g

report as 42.5 g

Division

$$4.203 \text{ m} / 0.0920 \text{ s} = 45.6847826087 \text{ m/s}$$

report as 45.7 m/s

Examples: Arithmetic Operations and Significant Figures (continued)

Multiplication

$$30.81 \text{ m} * 27 \text{ m} = 831.87 \text{ m}^2$$

report as 830 m^2

Note that the number of significant figures present in "830" is ambiguous - is the zero significant or not? It would be clearer to report the number as $8.3 \times 10^2 \text{ m}^2$ if you want to communicate that it has two significant figures.

Detailed Consideration of the Multiplication Example

The least certain multiplicand has two significant figures. The value 27 m is uncertain by 1 unit. It could be as low as 26 m or as high as 28 m. Consider these two possibilities:

$$30.81 \text{ m} * 26 \text{ m} = 801.06 \text{ m}^2 \quad \text{report as } 8.0 \times 10^2 \text{ m}^2$$

$$30.81 \text{ m} * 28 \text{ m} = 862.68 \text{ m}^2 \quad \text{report as } 8.6 \times 10^2 \text{ m}^2$$

The first uncertain digit in the product is in the tens place so the product ($30.81 \text{ m} * 27 \text{ m}$) is reported to only two significant digits, $8.3 \times 10^2 \text{ m}^2$. Notice that the product has the same number of significant figures as the least certain multiplicand.

Questions About the Examples

- When you add or subtract numbers, how do you identify the first uncertain digit in the result?
Look for the least precisely known # of those being added or subtracted & identify its first uncertain digit (last digit). The first uncertain digit in the result is in the same position w/ respect to the decimal.
- When you multiply or divide numbers, what is the relationship between the number of significant digits in the result and the number of significant figures in the numbers you are multiplying or dividing?

The number of s.f. in the result is equal to the number of s.f. in the least precisely known number.

- Explain, in a manner similar to that used for the **Detailed Consideration of the Multiplication Example**, why the result for $4.203 \text{ m} / 0.0920 \text{ s}$ should be reported only to 3 significant figures.

least precisely known $\Rightarrow 0.0919$ to 0.0921

Range of results:

$$\text{Low } \frac{4.203}{0.0921} = 45.63517... \quad \text{High } \frac{4.203}{0.0919} = 45.73449...$$

Only the "45" is certain. The tenths position is uncertain but significant.

Problems

- The mass of a coin was measured three times and each measurement was made to five digits. The mass values were 23.319 g , 23.341 g , and 23.296 g . The average mass was reported as 23.32 g . The true mass of the coin is 25.5631 g .

These don't agree well! If rounded a bit,

23.30
23.34
23.30

last digit is uncertain

- Are these measurements precise? Accurate? Explain your answers. Neither. The last repeatable digit is only the tenths position. The measured average differs from the true mass by $> 2 \text{ g}$!
- Though all three measurements were made to five digits, the student (correctly) decided to report the average mass to only four digits. Why was this a good idea?
You see that 1st digit that isn't the same for all measurements is the hundredths position. It's uncertain but it's significant.

- Report the total mass of three people weighing 53 kg , 60.4 kg , and 75.67 kg . Explain the rationale for the number of significant figures you report.

$$\begin{array}{r} 53. \\ 60.4 \\ 75.67 \\ \hline 189.07 \end{array} \quad \begin{array}{l} \leftarrow \text{least precisely known;} \\ \text{ones position is uncertain} \\ \Rightarrow \text{result is known to ones position} \end{array} \quad \Rightarrow \boxed{189 \text{ g}}$$

Something to Think About: In a multistep calculation, do you look only at the last step to determine the number of significant digits in the final result? No. To determine the s.f. in the final result, you must consider the numbers of s.f. in the numbers used to get that result. These depend on the s.f. in the previous steps. You must track the s.f. in each step.