Framework for Quantum Mechanics

Postulates

(encountered already - but not formalized quite)

One: The wavefunction, $\psi(\vec{x})$, completely describes the state of a system (e.g. particle or molecule)

[redacted] $\psi^*(x)\,\psi(x)\,dx$ is the probability the particle can be found between x & $x+dx$

(Born interpretation)

Two:

[redacted] For every observable there exists a linear quantum mechanical operator with real eigenvalues. (Hermetian)

[redacted] The only possible values of an observable are its eigenvalues.

[redacted] The average value of an operator, in the n^{th} state is given by

$$\langle \hat{O} \rangle = \langle n | \hat{O} | n \rangle$$

Three: The wavefns are solutions to $\hat{H}\psi(\vec{x},t) = i\hbar \frac{\partial}{\partial t}\psi(\vec{x},t)$

what's a linear operator?

satisfies $\hat{A}(f+g) = \hat{A}f + \hat{A}g$ and $\hat{A}(af) = a\hat{A}f$ when a is a constant

why do we care?

because when $\hat{A}$ is linear, then any linear combination of degenerate eigenfunctions is also an eigenfunction

so?? for example 2p orbitals are not all $\mathbb{R}$ functions, but this is inconvenient, using this theorem I can combine the complex fns to get real ones!

proof? if $\hat{A}f = af$ and $\hat{A}g = ag$ then

$\hat{A}(f+g) = a(f+g)$

$\boxed{\text{try this @ home, see posted soln}}$