Due: September 15th

Problem 1.

Bode’s Law (Eq. 1.3 in your Chapter 1 handout) gives the orbital radius of the \( n \)th planet from the Sun (counting the asteroid belt) in astronomical units. It fits the observations well except for Neptune \((n = 9)\) and Pluto \((n = 10)\). Calculate the orbital radii of Neptune and Pluto predicted by Bode’s Law, and compare the results with the observed values (Table 1.2 in your Chapter 1 handout). Express the discrepancies as percentages of the predicted distances.

Problem 2.

The barycenter of a star and its planet – or of a planet and its moon – is the CENTER of mass of the pair – meaning there is a perfect balance between the mass of the two objects and their respective distances to the barycenter. Using the mass and radius of primary body and satellite, and the orbital radius of the satellite, as given in Tables 1.1–1.3, calculate the location of the barycenter of the following pairs of bodies. In each case, does the barycenter lie inside or outside the primary body?
(a) Sun and Earth;
(b) Sun and Jupiter;
(c) Earth and Moon;
(d) Pluto (mass = \(1.27 \times 10^{22}\) kg; radius = 1137 km) and Charon (mass = \(1.9 \times 10^{21}\) kg; radius = 586 km); the radius of Charon’s orbit is 19,640 km.

Problem 3.

A planet with radius \( R \) has a mantle with uniform density \( \rho_m \) enclosing a core with radius \( r_c \) and uniform density \( \rho_c \). Show that the mean density of the planet, \( \rho \), is given by:

\[
\frac{\rho - \rho_m}{\rho_c - \rho_m} = \left( \frac{r_c}{R} \right)^3
\]

Hint - the solution requires expressing the separate masses of core and mantle.

Problem 4.

The radius of the Moon is 1738 km and its mean density is \(3347\) kg m\(^{-3}\). If the Moon has a core with radius 400 km and the uniform density of the overlying mantle is \(3300\) kg m\(^{-3}\),
what is the density of the core? How does this compare with the Earth’s core density? This is a variation on the previous question and the same formula can be assumed.

**Problem 5.**

Moment of inertia factors I/MR$^2$ (and other information) for the various planets can be found at [http://nssdc.gsfc.nasa.gov/planetary/planetfact.html](http://nssdc.gsfc.nasa.gov/planetary/planetfact.html). Find the values for the Sun, Venus, Mars, the moon, Earth, and Jupiter. Put these in order from large to small and explain what they tell about the different object’s density distributions (the way that the density changes from the surface to the center of an object).

**Problem 6.**

The moon has a nearly circular orbit around the Earth with a semimajor axis of approximately 384,400 km, an orbital period of 27.32 days and Earth's mass to be 5.97 x10$^{24}$ kg. Using a Newtonian modification of Kepler’s third law – replacing M with (M+m) – estimate the mass of the moon?

Using the Earth's mass and taking the radius of the moons orbit around the Earth to be 384.4 x10$^3$ km, what is the acceleration of the Moon toward the Earth? Recall that in order for an object to execute circular motion - even at a constant speed - the object must be accelerating towards the center of rotation. This acceleration is called the centripetal or radial acceleration ($a_c$) and is given by

\[ a_c = \frac{v^2}{r} \]

where v is velocity and r is orbital radius.

If the gravitational force exerted on a body by the Earth is inversely proportional to the square of the distance from the Earth’s center (as Newton showed), the acceleration produced by this force should vary in the same way. Therefore, if the acceleration of the moon is caused by the gravitational attraction of the Earth, the ratio of the moon’s acceleration to that of a falling body at the Earth’s surface should equal the ratio of the square of the Earth’s raius to the square of the radius of the moon’s orbit. Is this true?