

Natural Hazards - Geology 209
Homework assignment #5 - Volcanoes

Due Date: **March 5th**

Name _____

Magma Viscosity

1. To properly understand the hazards associated with lava flows from a volcanic source one must first understand the composition of the magmatic source in question (i.e. the composition of future eruptive material), the magmas density and ultimately the magmas viscosity.

a) Plot a graph of the velocity profile through a 2-m-thick flow of basalt (density = 2.75g cm⁻³) and a 2-m-thick flow of rhyolite (2.5 g cm⁻³). The basalt has a predetermined viscosity of 300 Pa·s and the rhyolite has a viscosity of 10⁵ Pa·s. Both lavas are descending a slope of approximately 5°.

To solve the velocity profiles use the following relationship:

$$v = \frac{\rho g \sin \theta}{\eta} \left(hz - \frac{z^2}{2} \right)$$

where v is velocity, ρ is density, g is the acceleration due to gravity (9.8 m/sec²), η is viscosity, θ is the slope of the terrane, h is the flow thickness and z is the elevation in the flow (i.e., the variable that you will need to change to calculate the velocity profiles).

b) Describe the shape of the profiles?

c) Estimate your own maximum running speed in the same units of velocity. Which flow would be easiest to outrun?

d) What is the maximum velocity within the basaltic and rhyolite flows?

e) What would happen to the flow rates if the same volcanic material were erupted on the moon? Why?

f) Do the flow velocities seem low, high or just right to you? Explain your reasoning, and tell how the estimates of velocity might be improved (i.e., what other information might be helpful to improve this calculation).

g) Calculate the maximum velocity within the flow for lunar basalt from the Sea of Tranquility having a viscosity of 9 Pa·s and a density of 3.0 g cm⁻³ (note that the acceleration due to gravity on the moon is only 1.62 m s⁻²).

Rising Magmas

2. Within a body of magma surrounded by denser rock, there is an excess pressure gradient, that is, there is an excess pressure gradient due to the lower density of the magma. Magmas are usually always less dense than their surrounding rock and thus total melting of a volume of rock results in buoyancy forces on the magma, which causes the magma to rise in the crust.

Let us consider the buoyancy rise of magma through a typical slice of continental crust from a source in the upper mantle. This crust is taken to consist of an upper 35 km with an average density of 2.70 g cm^{-3} and a lower 15 km with an average density of 3.00 g cm^{-3} . The magma was formed at a depth S beneath the surface in the upper mantle, which has a mean density of 3.20 g cm^{-3} (remember that the density of magma is about 90% that of the original rock). Let us suppose that this magma was injected into the earth along a fracture associated with continental rifting. At some point within the crust the magma will lose its density difference or it will rise all the way to the surface and erupt. Under this condition, the pressure at the base of the magma chamber is equal to that at the base of the adjoining rock column. Here is the mathematical relationship:

Pressure from total rock column above the original magma chamber = Pressure of an equally thick column of magma – Pressure of the rock column above where the magma chamber sits now

Recall that lithostatic Pressure (P) is:

$$P = \rho gh$$

where ρ is density, g is the acceleration due to gravity, and h is the thickness of material above. For depth below the surface h is positive, for height above the surface h is negative.

- a) What is the depth in meters from the earth's surface to the magma source if the magma rose all the way to the earth's surface along the fracture?
- b) What would be the position of the magma chamber pool if the starting depth was 70 km? What is the likelihood that this magma chamber erupts soon?

Magma Viscosity

3. Fractional crystallization is a process that typically involves the sinking or floating of crystals in a magma chamber. Gravity, acting on a contrast in density between the crystals and magma, is the driving force for crystal settling. The magma will resist crystal settling by exerting viscous forces on the crystals (resisting them to flow or move). Crystals may settle slowly or not at all in a viscous magma, whereas they will settle rapidly in magma of low viscosity (high fluidity). The settling velocity of an ideal, spherical crystal is described by Stokes equation:

$$V = \frac{2gr^2(\rho_{crystal} - \rho_{liquid})}{9\eta}$$

where:

V = velocity of settling (m s⁻¹)

g = acceleration of gravity (9.80 m s⁻²)

r = radius of spherical crystal (m)

$\rho_{crystal}$ = density of the crystal (kg m⁻³)

ρ_{liquid} = density of the liquid (kg m⁻³)

η = viscosity of the liquid (kg m⁻¹ s⁻¹) (note: 1 (kg m⁻¹ s⁻¹) = 10 poise)

As you can see from Stokes equation, there are quite a few variables that affect settling velocity. The settling velocity will be faster for larger crystals (large r) than for smaller crystals (small r). It will be faster for crystals with a density that is much greater than that of the liquid ($\rho_{crystal} - \rho_{liquid}$ is large). Settling velocity will be slower for liquids with a high viscosity. etc.

- (a) How long will it take an olivine crystal ($r = 1\text{mm}$, $\rho = 3.3 \times 10^3 \text{ kg m}^{-3}$) to settle to the bottom of a small magma chamber (100 m) in a basalt ($\rho = 2.7 \times 10^3 \text{ kg m}^{-3}$, $\eta = 20 \text{ kg m}^{-1} \text{ s}^{-1}$)?
- (b) How long will it take a quartz crystal ($r = 1\text{mm}$, $\rho = 2.7 \times 10^3 \text{ kg m}^{-3}$) to settle to the bottom of a small magma chamber (100 m) in a basalt ($\rho = 2.7 \times 10^3 \text{ kg m}^{-3}$, $\eta = 20 \text{ kg m}^{-1} \text{ s}^{-1}$)?
- (c) How do you think these different settling velocities affect the final igneous intrusive rock after it has crystallized entirely?

Volcanic Eruptions

4. In a volcanic eruption the tephra (a collective term for all material ejected from a volcano and transported through the air) reaches a maximum elevation (h_0) of 10000 feet from the vent of the volcano.

- a) What was the initial velocity of the tephra as it was ejected from the volcano (assume the elevation of the volcano is 2500 meters)? To find the velocity use the kinetic energy equation and set the kinetic energy of the tephra as it explodes equal to the total potential energy the tephra has as it stops at its maximum elevation.

$$KE = (1/2)mv^2$$

$$PE = mgh$$

- b) Once the tephra starts flowing down the side of the volcano, it will start to lose its total energy through friction as a function of the horizontal distance (x) away from its starting point. The total energy, as a function of horizontal distance, decreases as:

$$E_{\text{total}}(x) = mg(h_0 - Ax) = KE + PE$$

where x is the horizontal distance away from the summit, and A is the friction coefficient, $A = 0.2$. What is the maximum horizontal distance the tephra can travel? (Hint: the maximum distance is reached when the tephra's velocity is zero).