Parametric Equations and Systems of Differential Equations

GROUP MEMBERS:

1. _____
2. _____
3. _____
4. _____

Each person in the group, write down your name. In this group exercise, each person in the group will have a job.
Person 1: The manager - manages the group, ensures that members are fulfilling their roles, that all members of the group participate and understand the concepts. The instructor responds only to questions from the manager.
Person 2: The recorder - records the group data and explanations.
Person 3: Grapher - plots the graphs for the group.
Person 4: Presenter - presents oral reports to the class when requested by the instructor and reads the instructions to the group.

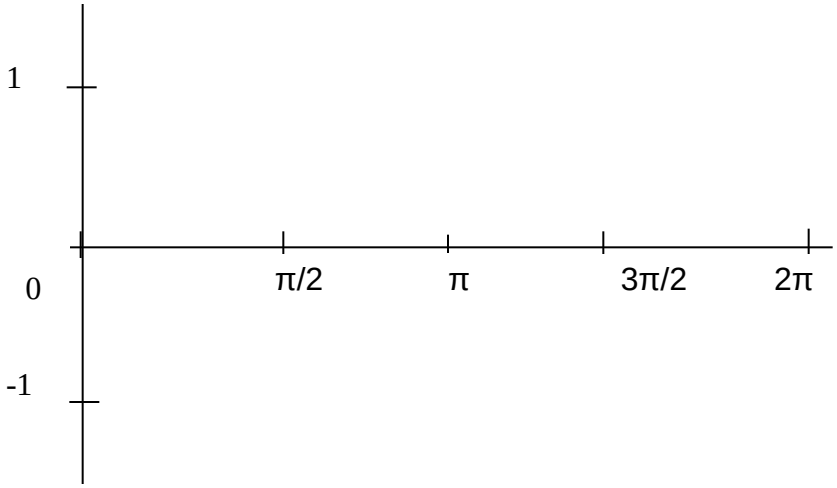
Parametric Equations: Goal - to graph the parametric equations
$(x(t) = \cos t, y(t) = \sin t)$ for $t \in [0, 2\pi]$.

1. First fill in the following table listing the values of t, x(t), y(t)

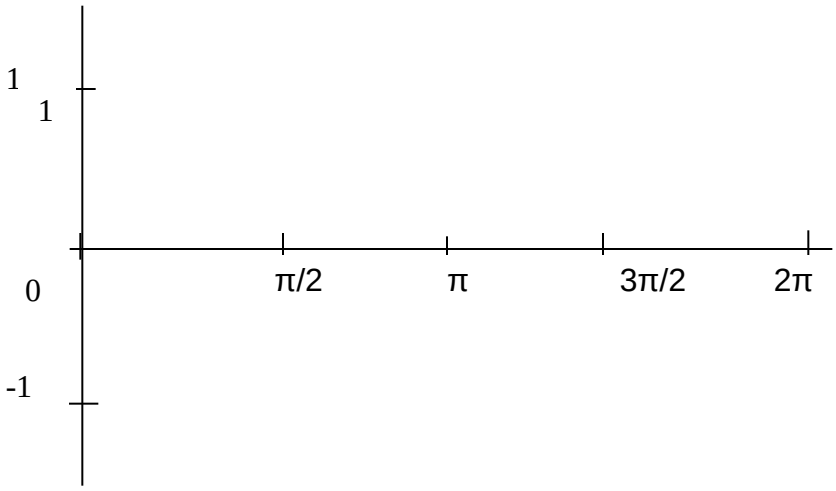
t	x(t) =cos(t)	y(t)=sin(t)
0		
π/4		
π/2		
3π/4		
π		
5π/4		
3π/2		
7π/4		
2π		

2 Graph these functions in different ways.

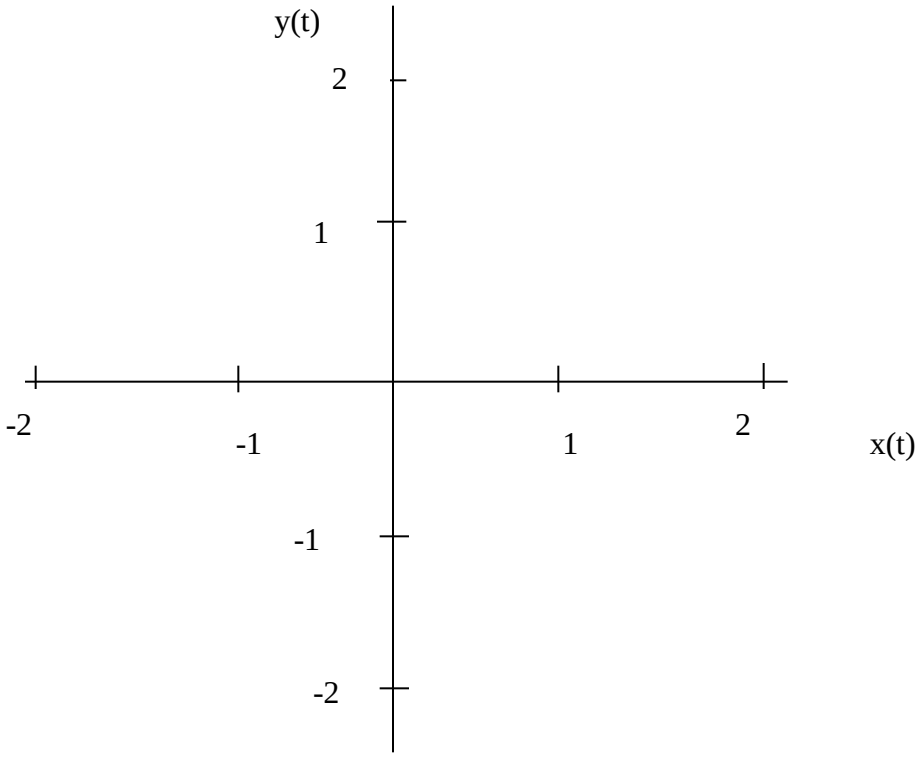
a. Graph cos(t) as a function of t. Graph t on the horizontal axis, cos(t) on the vertical axis.



b. Graph sin(t) as a function of t. Graph t on the horizontal axis, sin(t) on the vertical axis.



c. Graph (x(t) = cos(t), y(t) = sin(t)), with x(t) on the horizontal axis and y(t) on the vertical. Start by plotting the points (x(t), y(t)) for the values of t in the table above and then guess where the other (x(t), y(t)) values would go.



d. Show that the parametric curve (x(t) = cos(t), y(t) = sin(t)) satisfies the equation

$x^2 + y^2 = 1$.

Hence what type of curve does this curve define?

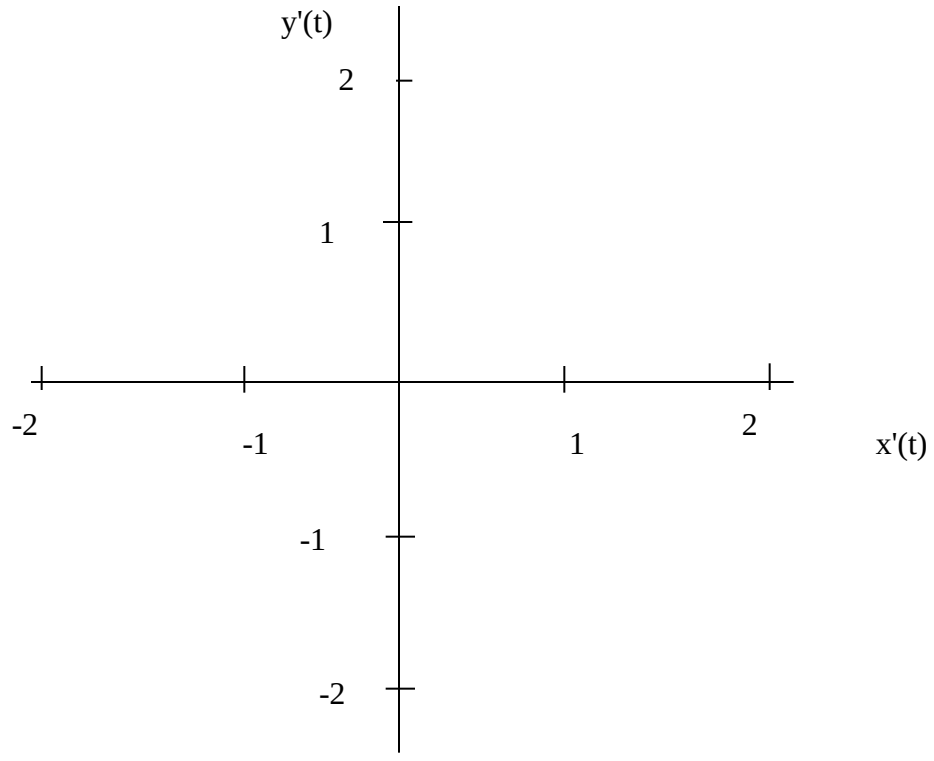
2. Tangent Vectors. The tangent vector to a curve (x(t) , y(t)) is defined to be the vector v(t)=(x'(t) , y'(t)).

a. Calculate: x'(t) = y'(t) =

b. Fill in the following table.

t	x'(t) =	y'(t)=
0		
π/4		
π/2		
3π/4		
π		
5π/4		
3π/2		
7π/4		
2π		

c. For each of the t values, graph the vector v(t) =(x'(t) , y'(t)) on the axis below. Start each vector at the origin. Label each vector with its t value.



d. The more usual way to draw the tangent vectors v(t) is to move the vector from the origin and attach the vector to the curve (x(t), y(t)) at the appropriate point. Go back to (1c) and for each t value in the table, go the point (x(t), y(t)) and attach at that point the vector v(t) = (x'(t), y'(t)). At each point (x(t),y(t)), draw new xy axes (very faintly) centered at that point. Then plot the vector v(t) relative to those axes.

3. Systems of Differential Equations.

a. Show that the curves (x(t) = cos(t), y(t) = sin(t)) satisfy the system of 2 differential equations:

$$x' = y$$
$$y' = x$$

b. Is this system of differential equations linear or non-linear? Explain.

c. Vector Field and Phase Space.
Given the

Person	x	y	dx/dt	dy/dt
1	0	0		
2	1	0		
3	1	1		
4	0	1		
1	-1	1		
2	-1	0		
3	-1	-1		
4	0	-1		
1	1	-1		