Section 4.1 – Using the First and Second Derivatives

Definitions. Let $f$ be a function.

1. A critical point of $f$ is a point $p$ in the domain of $f$ such that either $f'(p) = 0$ or $f'(p)$ is undefined.
2. We say that $f$ has a local minimum at $p$ if $f(p)$ is less than or equal to the values of $f$ for points near $p$.
3. We say that $f$ has a local maximum at $p$ if $f(p)$ is greater than or equal to the values of $f$ for points near $p$.
4. An inflection point of $f$ is a point at which the function $f$ changes concavity.

Example. Given to the right is the graph of a function $f$.

(a) Estimate the critical point(s) of $f$.

(b) Estimate the inflection point(s) of $f$.

(c) Does $f$ have any local maximum or local minimum values? If so, list them, making it clear which are which.

First Derivative Test. Suppose that $p$ is a critical point of a continuous function $f$.

1. If $f'$ changes from negative to positive at $p$, then $f$ has a ______________________ at $x = p$.
2. If $f'$ changes from positive to negative at $p$, then $f$ has a ______________________ at $x = p$.

Second Derivative Test.

1. If $f'(p) = 0$ and $f''(p) > 0$, then $f$ has a ______________________ at $x = p$.
2. If $f'(p) = 0$ and $f''(p) < 0$, then $f$ has a ______________________ at $x = p$. 

Activities to accompany Calculus, Hughes-Hallett et al, Wiley, 2013
EXERCISES.

1. Let \( f(x) = x^{2/3}(4 - x)^{1/3} \).

   (a) Given that \( f'(x) = \frac{8 - 3x}{3x^{1/3}(4 - x)^{2/3}} \), find the intervals on which \( f \) is increasing/decreasing.
(b) Given that $f''(x) = \frac{-32}{9x^{4/3}(4-x)^{5/3}}$, find the intervals on which $f$ is concave up/concave down.
(c) Find all local maxima, local minima, and inflection points of $f$. 
2. Given to the right is the graph of the DERIVATIVE of a function. Use this graph to help you answer the following questions about the ORIGINAL FUNCTION $f$.

(a) What are the critical points of $f$?

(b) Where is $f$ increasing? decreasing?

(c) Does $f$ have any local maxima? If so, where?

(d) Does $f$ have any local minima? If so, where

(e) Where is $f$ concave up? concave down?
3. Given to the right is the graph of the SECOND DERIVATIVE of a function. Use this graph to help you answer the following questions about the ORIGINAL FUNCTION $f$.
   
   (a) Where is $f$ concave up? concave down?

(b) Does $f$ have any inflection points? If so, where?