Bifurcations

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Group Members:

1. __________________________
2. __________________________
3. __________________________
4. __________________________

**Goal:** To study the effect of various levels of harvesting on the steady state fish population

\[
\frac{dP}{dt} = P(1 - P) - C = -P^2 + P - C
\]

gotten from including a fishing term in the logistic equation.
The value of C denotes how many fished per year are caught. When C = 0, no fish are caught and
the model reduces to the standard logistic model

\[
\frac{dP}{dt} = kP(1 - \frac{P}{N})
\]

with k=1 and N =1.

**Directions:** Each person in the group will examine the equation for a different value of C. The
group will break up and we will re-group people together who are working with the same C values.

Person 1: C = 0, C=.25
Person 2: C = .05, C=.4
Person 3: C = 1/8, C = .3
Person 4: C = .2, C = .5
When you return to your group, you will combine all your results into one picture. For each $c$ value, you will plot the phase line for that $c$ value as a vertical line in the $(c, y)$ plane. The union of all these phase line pictures will help us understand how the fish populations vary as we vary the harvesting amount $C$.

Go in order of increasing $c$ values. One at a time, each person adds their phase line picture to the overall picture. Explain what equilibrium points you calculated and describe their type.
Group Questions:
1. What happens to the fish population over the long term if the fishing level $c$ is high? Can you explain in ‘intuitive terms’ why this happens?

2. What is the highest fishing level you can have (ie. $C$ value) and still have a sustainable fish population?

3. If the harvesting level is below this critical level, what will happen to the fish population over time?
5. On your diagram, you have only filled in a few $c$ values. For a general value of $c$ (i.e. using the algebraic expression and not a numerical value), find the equilibrium solutions of the differential equation

$$\frac{dP}{dt} = P(1 - P) - C = -P^2 + P - C$$

6. Graph the $f(P) = -P^2 + P - C$ function for a general value of $C$. Make the zeros of the function agree with your calculation in (5).

7. Draw the general phase line and label the general equilibrium values.

8. Use this information to fill in the rest of the $(C, P)$ diagram for all values of $C$. 