Phase Lines

GROUP MEMBERS
1. ___________________________
2. ___________________________
3. ___________________________
4. ___________________________

Directions: Work together as a team.

Goal: To determine the phase line picture for an autonomous differential equation.

Consider the differential equation \( \frac{dy}{dt} = y^2 - 6y + 5 \).

1. Is the constant function \( y(t) = 2 \) a (equilibrium) solution of this differential equation? Justify.

2. Is the constant function \( y(t) = 5 \) a (equilibrium) solution of this differential equation? Justify.

3. Give all equilibrium solutions for this differential equation. Explain your reasoning.
4. Choose five \( y \) values and calculate the slope fields for these \( y \) values. Be strategic in your choice of \( y \) values. You want to pick the \( y \) values that will best help you determine the behavior of solutions of the differential equation. You may do preliminary calculations to help you decide which \( y \) values to choose.

Preliminary work:

<table>
<thead>
<tr>
<th>Choice of ( y ) values</th>
<th>Value of slope at points ( (t = -2, y) )</th>
<th>Value of slope at points ( (t = 0, y) )</th>
<th>Value of slope at points ( (t = 2, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1 = )</td>
<td></td>
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<td></td>
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<tr>
<td>( y_2 = )</td>
<td></td>
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<tr>
<td>( y_3 = )</td>
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<td>( y_4 = )</td>
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<tr>
<td>( y_5 = )</td>
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</table>

What was your strategy in choosing these \( y \) values?

5. Sketch the slope field at these 15 points on the \((t, y)\) plane.

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Phase Line
6. On the (t, y) plane, sketch solution curves through a variety of initial starting points \((t_0, y_0)\) until you feel that you have a complete understanding of how solutions behave.

7. For what values is the slope of the slope field negative? For what values is the slope of the slope field positive?

8. Draw a sketch of the slope function \(f(y)\) in the \(y-f(y)\) plane. How does the graph of the slope function \(f(y)\) relate to your slope field in (5).

9. Draw the phase line diagram to the left of the (t,y) plane picture. The idea is to project the two dimensional \((t, y)\) picture onto a one dimensional picture just on the y-axis. Put a dot at the y-values that are equilibrium solutions. Put an up arrow if the solutions are increasing in a certain region. Put a down arrow if the solutions are decreasing. Play with the DETools program **Phase Lines** to get a feeling for what the phase line shows.