I. Section 1.2 # 15. Find the solution of the logistic differential equation

\[ \frac{dy}{dt} = y(1 - y) \]

so that here \( k = 1 \) and \( N = 1 \). This is a challenging multi-step problem; I outline the steps you should take.

a. Re-write \( \frac{1}{y(1-y)} \) as

\[ \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y}. \]

To determine the values of A and B, get a common denominator on both sides, set the numerators on both sides equal to each other. Since the two sides must be equal for all values of \( y \), choose some simple values of \( y \) that will make it easy for you to figure out the values of A and B. This technique of splitting a fraction apart into two pieces, which is the opposite of getting a common denominator, is called the Method of Partial Fractions.

b. Show that

\[ \int \frac{1}{y(1-y)} \, dy = \int \frac{A}{y} + \frac{B}{1-y} \, dy = \ln |\frac{y}{1-y}| \]

once you plug in the values of A and B.

c. Conclude that

\[ \frac{y}{1-y} = c_1 e^t \]

where \( c_1 \) is any positive constant and hence that \( \frac{y}{1-y} = ke^t \) where \( k \) can be any real number.

d. Solve for \( y(t) \) and show that

\[ y(t) = \frac{ke^t}{1 + ke^t}. \]

Section 1.5 #2,

# 9 In addition to parts (a, b, c), show: (d) calculate \( \partial f/\partial y \) and show that the differential equation satisfies the conditions of the Uniqueness Theorem.

# 14. In addition to parts a, b, c from the book, do the following: (d) For this solution, what is the largest value that the \( \epsilon \) given in the existence theorem can take on? (e). Find the solution of this differential equation that satisfies the initial condition \( y(0) = 10 \). How long does this solution exist before "blow up" occurs?

Sect 1. 9 #24 - Write the differential equation that models the situation described. What is the initial condition? Can this equation be solved using the separation of variables technique? Why or why not? **Do not solve this differential equation.**

Sect 1.4 #2 (do by hand), 6 (do using Excel), 9 (do using Excel).