Review Questions for Final Exam:
Questions that were challenging on Midterm 2:

7. (15 pts). The temperature $T(t)$ of a cup of coffee that is placed in a room whose temperature is 75 degrees Fahrenheit is modeled by the differential equation:

$$\frac{dT}{dt} = \frac{1}{4}(75 - T)$$

where $t$ = time in minutes.

a. Determine the solution of this differential equation. (You need to use $u$ and $du$. Minus sign!!).

b. If the temperature of the cup is initially 160 degrees, what is the temperature of the cup 10 minutes later?

c. Graph the temperature function $T(t)$ that you get.

3d. Use the Integral Table to determine $\int \frac{6}{(9-4x^2)^{3/2}} \, dx$.

Sequences:

1a. For the sequence $s_k = 2 + (-1)^k 4k$, $k = 0, 1, 2, 3, \ldots$, write out the first 4 terms of the sequence $s_1, s_2, s_3, s_4$. Is this sequence monotone increasing or monotone decreasing, is it oscillating, is it bounded? Does this sequence have a limit?

b. Give a formula for the nth term of the sequence that starts with 2, $-\frac{7}{2}$, $\frac{12}{4}$, $-\frac{17}{8}$, $\frac{22}{16}$, $\ldots$.

c. Give an example of a monotone increasing sequence that is bounded. Given an example of a monotone increasing sequence that is unbounded. What does one know about the convergence of each of these types of sequences?

d. Determine if the following sequences have a limit and what the limit is.

$$\lim_{n \to \infty} \frac{3 + 5n^2}{n + n^2} \quad \lim_{n \to \infty} 3 - \frac{2}{n} \quad \lim_{n \to \infty} \frac{n^2}{e^n} = \lim_{x \to \infty} \frac{x^2}{e^x}$$

Use the limit laws to find the following limit:

$$\lim_{n \to \infty} \{(3 + 5n^2) + (3 - \frac{2}{n})\}$$

Series:

2a. Given a series $\sum_{k=1}^{\infty} a_k$, how does one determine if the series converges or diverges? If the series converges, what is its value?

b. Write out the terms of the following series. Determine the partial sums $s_1, s_2, s_3, s_4$.

$$\sum_{k=1}^{\infty} \frac{1}{2^k}$$
Use the formula for the partial sum of a geometric series to give the value of $s_n$.

In this example, what does $\lim_{n \to \infty} s_n$ equal?

c. Re-write the following series using summation notation.

$$1 - \frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \ldots$$

$$3 + \frac{9}{4} + \frac{27}{16} + \frac{81}{64} + \ldots$$

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots$$

For each of these series, determine if it converges or diverges. Justify why it converges or diverges by referring to one of our tests. If it converges, give the value of the series. Which, if any, of these series are geometric series?

3.a. Integral Test. Use the integral test to determine if the following series converges or diverges.

$$\sum_{k=1}^{\infty} \frac{1}{(3k - 1)^{3/2}}$$

Draw a picture that relates the sum to the integral $\int_1^{\infty} \frac{1}{(3x - 1)^{3/2}} \, dx$ and explain.

b. Determine if the following series converge or diverge and justify.

$$\sum_{k=1}^{\infty} \frac{1}{(k)^7} \quad \sum_{k=1}^{\infty} \frac{1}{(k)^{7/3}} \quad \sum_{k=1}^{\infty} \frac{5 + k}{(k)^{7/3}} \quad \sum_{k=1}^{\infty} \frac{(7)^k}{k}$$

4. Power Series

a. Determine for what values of $x$ the following power series converge. What is the radius of convergence of the power series? If possible, determine the value of the series.

$$\sum_{k=10}^{\infty} \frac{x^k}{3^k} \quad \sum_{k=1}^{\infty} \frac{x^k}{k!} \quad \sum_{k=1}^{\infty} \frac{(x - 7)^k}{k}$$

5. Taylor Series.

a. Give the general formula for a Taylor Series for the function $f(x)$ based at the point $x_0$.

b. Calculate the Taylor Series for the function $f(x) = \sin(x)$ (or $\cos(x)$) based at $x_0 = 0$.

c. Determine for which values of $x$ this series converges. What is the radius of convergence of this Taylor Series?