Quiz Week 6 Redo
Math 102, Spring 2009

Name:

1. The half-life of cesium-137 is 30 years. Suppose we have a 30 mg sample.
   a. Find the mass that remains after t years. 
      \[ m(t) = ce^{kt} \]. Since \( m(0) = 30 \text{mg} \), \( m(t) = 30e^{kt} \). The half life calculation 
      \( m(30) = 15 = 30e^{k30} \) which gives that \( \ln(1/2) = k30 \) and hence 
      \[ k = -\ln(2)/30 = -0.0231. \]
      
      \[ m(t) = 30e^{-0.0231t}. \]
   b. How much of the mass will remain 50 years?
      \[ m(50) = 30e^{-0.023150} = 9.449. \]
   c. How long will it take till only 5 mg remains?
      \[ m(t) = 30e^{-0.0231t} = 5 \] implies that \( e^{-0.0231t} = 5/30 = 1/6 \) so that 
      \( -0.0231t = \ln(1/6) = -\ln 6. \) Finally 
      \[ t = -\ln 6/(-0.0231) = 77.56 \text{ years}. \]

2. Translate the following sentence into a differential equation. State clearly what your 
   variables represent.
   a. Water is flowing out of a bathtub. The rate at which the water is flowing out is proportional to the amount of water in the tub.
      Let \( w(t) \) be the amount of water in the bathtub after \( t \) seconds. Then 
      \[ \frac{d}{dt} w = kw. \]