Math 301 Homework
Donnay, Fall 2011

Due Thursday, Sept 8 or Friday Sept 9th at the start of class.

S1. For each of the following, explain your answer (unless the result is “obvious”). These can be examples from everyday life.

a. Make up a statement of the form: "If A then B" that is true.

b. Make up a statement of the form: "If A then B" that is false.

c. Make up a statement of the form: "If A then B" that is true but its converse is false.

d. Make up a statement of the form: "If A then B" that is true and whose converse is true. Then rewrite the statement in the form " A if and only if B”.

Morgan, Ch. 1 #1ab, 2ab, 4(explain your answer), 9abcd (draw graphs of the functions and use the graphs to help answer the question).

S2. a. Find all the fixed points for the iteration/dynamical system given by \( x_{n+1} = f(x_n) \) with \( f(x) = 2x(1-x) \). Recall that a fixed point is a solution of the equation \( f(x) = x \). Solve this problem using algebra.

b. Using the results from (a), determine if the following statements are true or false where \( f(x) = 2x(1-x) \).

   If \( x=0 \) then \( x \) is a fixed point of \( f \).

   If \( x \) is a fixed point of \( f \) then \( x =0 \).

   \( x \) is a fixed point of \( f \) if and only if \( x =0 \).

c. A fixed point is said to be attracting if points that start near the fixed point approach the fixed point. A fixed point is repelling if points that start near the fixed point move away from the fixed point. For the fixed point(s) you found in (a), investigate if they are attracting or repelling. Do this numerically and graphically using Excel. Take several initial points near the fixed point, and determine if the iterates approach or move away from the fixed point. Make a table of your values in Excel for \( \{x_0, x_1, \ldots, x_{10}\} \). For each initial value, state what happens to the iterates. Be sure to summarize your final conclusion as to whether the evidence suggests that the fixed point is attracting or repelling.

In addition to summarizing your results (using full sentences), hand in a copy of the Excel table that contains the iterates. Hand in an Excel graph showing the iterates for one of the initial values.

S3. a. Find all the fixed points for the iteration/dynamical system given by \( x_{n+1} = f(x_n) \) with \( f(x) = 4x(1-x) \).
b. Choose several different initial points that lie in the unit interval \((0, 1) = \{ x \in R : 0 < x < 1 \}\). Do the iterates approach a fixed point? Describe the behaviour of the iterates. Use Excel. Take at least 20 iterates.

c. In addition to summarizing your results (using full sentences), hand in a copy of the Excel table that contains the iterates. Hand in an Excel graph showing the iterates for one of the initial values.

S4. Make a conjecture for the general formula for the nth iterate \(x_n\) when the initial point is given as \(x_0\) and the function is \(f(x) = \frac{1}{2}x\).

“What to do when you do not know what to do”. What strategies did you use to figure this out?