Math 301 Homework
Donnay, Fall 2011

Part I: Due Wednesday, Sept 21 by 5pm. Part II is due Friday Sept 23th by 5pm. Put your work into the appropriate folder in the bin outside Professor Donnay’s office or hand the homework in during class.

Part I
S1. In class we proved the following. Theorem: Let $A$ and $B$ be sets with $A \subset B$. If $B$ is countable then $A$ is countable.
   a. State the converse of this theorem. Is the converse true (give a proof) or false (give a counter-example).
   b. State the contraposition of this theorem. Is the contraposition true or false?
   c. Use the result of (b) to give an example of an uncountable set. You may assume that the closed unit interval $I = [0, 1]$ is uncountable (we proved that in class). Try to make up a set that is not exactly the same as the ones we have seen in class already.

S2. Finish the follow definition: A sequence $s_n$ converges to a limit $L$ if . . . .
Study the definition but then close your books and try to write out the definition from memory. You will be asked for the definition on this week’s quiz.
Morgan, Ch 3, #1, 4, 5, 6 (do not need to do rigorous proofs).

Morgan, Ch 3, # 10. Use your calculator (or Excel) to generate a data table. Make a conjecture as to what the limit seems to be. Then practice using tolerance value and waiting time. For $\epsilon = .3$, determine the waiting time $N$. For $\epsilon = .05$ determine the waiting time.

S3. Predict the limit $L$ of the sequence $s_n = 2 + \frac{1}{n^2}$. First determine $N$ when $\epsilon = .1$. Then determine $N = N(\epsilon)$ when $\epsilon = .0001$ so that $s_n$ is very, very close to $L$!! Do this via algebra rather than using a calculator.

Part II
S1. Find an example of a fractal shape on the web. Post the web link and a brief description of the example in the course Blackboard site. Add your entry to the Discussion Board: Fractal. See my entry as an example.
S2. Give examples of the following types of sequences. Give the formula for $s_n$. Draw a two dimensional picture of the sequence.

a. Increasing, bounded, that converges to -3.
b. Oscillating, bounded that converges to 4.
c. Oscillating, bounded that diverges.
d. Increasing, unbounded that diverges.
e. Decreasing, bounded, that converges to 3.

S3. a. Predict the limit $L$ of the sequence $s_n = 2 + \frac{1}{n^2}$. Then use the formal $\epsilon - N$ definition of limit to give a rigorous proof of the limit.

b. For the sequence $s_n = 3 + 2^{-n}$, decide what the limit is. Then use the $\epsilon - N$ definition of limit to give a rigorous proof of the limit. First predict the limit of the sequence $s_n = 2 + (-1)^n \frac{1}{n}$. Then use the formal definition of limit to prove that this value is the limit.

S4. (Note: this problem does not have to be done ”rigorously”. We will return to these types of examples a bit later in the semester. This is a warm up exercise).

a. Define the sets $S_n = \left[\frac{1}{n}, 1 - \frac{1}{n}\right]$ for $n \geq 2$. Determine $\bigcup_{n=2}^{\infty} S_n$. Draw some pictures to help you understand the sets.

b. Define the sets $T_n = (1 - \frac{1}{n}, 1 + \frac{1}{n})$ for $n \geq 2$. Determine $\bigcap_{n=2}^{\infty} T_n$. 