Part I

S1. In class we proved the following. Theorem: Let $A$ and $B$ be sets with $A \subseteq B$. If $B$ is countable then $A$ is countable.

(a) State the converse of this theorem. Is the converse true (give a proof) or false (give a counter-example).

(b) State the contraposition of this theorem. Is the contraposition true or false?

(c) Use the result of (b) to give an example of an uncountable set. You may assume that the closed unit interval $I = [0,1]$ is uncountable (we proved that in class). Try to make up a set that is not exactly the same as the ones we have seen in class already.

S2. Equivalence Relation. a. For the usual relation $\leq$ between real numbers (ex. $2 \leq 3$, $-4 \leq 1$), determine whether $\leq$ satisfies the three required properties of an equivalence relation. For each property, decide whether $\leq$ satisfies it or not. Just use your common sense knowledge about numbers; no formal proof required.

(b) Real World Connection: Give an example from the everyday world (i.e. non-mathematical) of a notion of “sameness”. For example, “two human beings are the same if they have same the same gender”. Then check whether your notion of sameness satisfies the three properties of equivalence relation or not.

S3. Finish the follow definition: A sequence $s_n$ converges to a limit $L$ if . . . .

Study the definition but then close your books and try to write out the definition from memory. You will be asked for the definition on this week’s quiz.

Morgan, Ch 3, #1, 4, 5, 6 (do not need to do rigorous proofs).

Morgan, Ch 3, # 10 Use your calculator (or Excel) to generate a data table. Make a conjecture as to what the limit seems to be. Then practice using tolerance value and waiting time.

(a) For $\epsilon = .3$, determine from your data the waiting time $N$. Draw both the two dimensional and one dimensional diagrams to illustrate this result.

(b) For $\epsilon = .05$ determine the waiting time $N$ from your data.
Hand in a copy of your data (i.e. all the values of \( n \) and \( s_n \) that you generated to help answer this question). Go up to \( s_{25} \) and list your values to three decimal places.

S3. Predict the limit \( L \) of the sequence \( s_n = 2 + \frac{1}{n^2} \). Determine \( N \) when \( \epsilon = .1 \). Do this via algebra rather than using a calculator.

Part II

S1. Find an example of a fractal shape or some other type of interesting mathematical shape on the web. Post the web link and a brief description of the example in the course Blackboard site. Add your entry to the Discussion Board: Fractal. See my entry as an example.

S2. Equivalence relation. Show that ‘same cardinality’ satisfies transitivity, the third property of equivalence relation. This will finish the proof that cardinality is an equivalence relation between sets. Under the “what to do when you do not know what to do” - start out by drawing a picture that illustrates the general situation described: sets A, B, C and functions between various of the sets. Remind yourself of the relevant definitions by re-writing them: B has the same cardinality as A, which we write as \( A \approx B \) if there exists a one-to-one and onto function \( f : A \to B \).

S2. Give examples of the following types of sequences. Give the formula for \( s_n \). Draw a two dimensional picture of the sequence.

a. Increasing, bounded, that converges to -3.

b. Oscillating, bounded that converges to 4.

c. Oscillating, bounded that diverges.

d. Increasing, unbounded that diverges.

e. Decreasing, bounded, that converges to 3.

S3. a. Predict the limit \( L \) of the sequence \( s_n = 2 + \frac{1}{n^2} \). Then use the formal \( \epsilon - N \) definition of limit to give a rigorous proof of the limit.

b. For the sequence \( s_n = 3 + 2^{-n} \), decide what the limit is. Then use the \( \epsilon - N \) definition of limit to give a rigorous proof of the limit.

S4. (Note: this problem does not have to be done "rigorously". We will return to these types of examples a bit later in the semester. This is a warm up exercise).

a. Define the sets \( S_n = [\frac{1}{n}, 1 - \frac{1}{n}] \) for \( n \geq 2 \). Determine \( \bigcup_{n=2}^{\infty} S_n \). Draw some pictures to help you understand the sets.

b. Define the sets \( T_n = (1 - \frac{1}{n}, 1 + \frac{1}{n}) \) for \( n \geq 2 \). Determine \( \bigcap_{n=2}^{\infty} T_n \).