In a dynamical system, something moves according to a rule. Examples include the planets which move according to the rules of gravity; a spring that moves according to the rules of spring motion (the restoring force is proportional to the displacement: Hooke’s law).

In a mathematical dynamical system, the something that moves are points on the number line and the rule of motion is given by a function.

As you work through the following examples, keep track of any questions, thoughts, conjectures that come to mind. A key part of mathematics is this process of questioning, wondering, and looking for patterns.

Examples:

1. Let \( f(x) = \frac{1}{2}x \). Take the starting point to be \( x_0 = 8 \). Then the new point is \( f(x_0) = f(8) = 4 \). We denote this new point by \( x_1 \) since it is gotten from \( x_0 \) by applying the rule of motion once: \( x_1 = f(x_0) = 4 \).

Calculate the following values:

\[
x_2 = f(x_1) = \quad , \quad x_3 = f(x_2) = \quad , \quad x_4 = f(x_3) = \quad 
\]

Continue in this way until you have gotten \( x_{10} \). Write down all your values.
Make some type of diagram/drawing that shows the motion of the points under the rule.

Can you see a pattern? What would you predict would happen if you continued applying the rule more and more times?

If the numbers you are generating represent (in units of hundred thousands) the population of polar bears in succeeding years (starting say in 1997 with 8 hundred thousand), what does the model predict for the polar bear population in 2000? in 2005? What does the model will happen to the polar bear population in the long term?

Reflections: Record any questions, thoughts, conjectures that you have related to this problem.
2. Take the dynamical system given by the rule \( f(x) = \sin(x) \). Choose any starting point you want for your value of \( x_0 \). Use your calculator to determine the successive values of \( x \). Make sure you are using radians. Record your values in a table up until \( x_{10} \). Calculator iteration: enter your initial value. Hit the sin button to get \( x_1 \). Then hit the sin button again to get \( x_2 \). Repeat.

After \( x_{10} \), keep repeating the rule (iterating) although you no longer need to write down every value. Look for a pattern. Record the value for \( x_{30} \) - if you can keep track :) . What seems to be happening to the values?

If these dynamical system were again a model of polar bear population (measured in units again of hundred thousands), what does the model predict about the population?

Reflections: Record any questions, thoughts, conjectures that you have related to this problem.
3. Repeat question (2), but now using the function $f(x) = \cos x$. 