Assignment #3: Due Wednesday, February 3 at the start of class.

S.1. Let \( f(x) = |x| \) so that \( f(x) = x \) for \( x > 0 \) and \( f(x) = -x \) for \( x \leq 0 \).
(a) Use the definition of derivative to show that \( f'(0) \) does not exist. Calculate
\[
\lim_{h \to 0^+} \frac{f(0 + h) - f(0)}{h} \quad \text{and} \quad \lim_{h \to 0^-} \frac{f(0 + h) - f(0)}{h}.
\]
You must replace the absolute value by the value of the function (either \( \pm x \)) when you work out your calculations. Illustrate your answers with diagrams showing each of cases.

(b) Again using the definition of derivative, show that \( f'(1) \) does exist. Calculate
\[
\lim_{h \to 0^+} \frac{f(1 + h) - f(1)}{h} \quad \text{and} \quad \lim_{h \to 0^-} \frac{f(1 + h) - f(1)}{h}.
\]
Illustrate your answers with diagrams showing each of cases.

S.2. Give an example of a function \( f(x) \) that is in the space \( C^3[-\infty, +\infty] \) but is not in \( C^4[-\infty, +\infty] \). Justify your answer. Draw graphs of \( f(x) \) and its derivatives to illustrate your discussion.

Assignment #4: Due Friday, Feb 5 at the start of class.

S.1. Algorithm for finding the maximum of a function \( f(x) \) defined on a closed, bounded interval \([a, b] \):
(a) Find the critical points of \( f \). These are points at which either \( f'(x) = 0 \) or at which \( f'(x) \) does not exist. Evaluate \( f \) at this points.
(b) Evaluate \( f \) at the endpoints: \( f(a), f(b) \).
(c) Compare these values. The largest will be the maximum of \( f \) on \([a,b]\). (The smallest will be the minimum).

Justify why this method works to find the maximum of the function. For example, how do we even know that the function has a maximum? State clearly what conditions you are assuming that the function \( f \) satisfies.

Morgan: CH. 28. # 3 Try to do this without looking at your notes. But if after a while you can not do it, then look back at the proof of the similar result for \( R \) in your notes or in the text book and modify the proof for a metric space.

Ch. 14 # 1,2