Comparing Improper Integrals

Name:

Goal: Comparing improper integrals to determine convergence or divergence.

Consider three positive functions that for \( x > 1 \) satisfy \( h(x) < f(x) < g(x) \).

I. Suppose the \( f(x) \) satisfies:

\[
\int_{1}^{\infty} f(x) \, dx = \lim_{t \to \infty} \int_{1}^{t} f(x) \, dx
\]

diverges (to infinity). Hence the area under the \( f(x) \) curve is infinite.

a. With \( g(x) > f(x) \) we have that \( \int_{1}^{\infty} g(x) \, dx \) converges diverges can not tell

b. With \( h(x) < f(x) \) we have that \( \int_{1}^{\infty} h(x) \, dx \) converges diverges can not tell

II. Suppose the \( f(x) \) satisfies:

\[
\int_{1}^{\infty} f(x) \, dx = \lim_{t \to \infty} \int_{1}^{t} f(x) \, dx
\]

converges (to a finite amount). Hence the area under the \( f(x) \) curve is finite.

a. With \( g(x) > f(x) \) we have that \( \int_{1}^{\infty} g(x) \, dx \) converges diverges can not tell

b. With \( h(x) < f(x) \) we have that \( \int_{1}^{\infty} h(x) \, dx \) converges diverges can not tell