Line Integrals

GROUP MEMBERS:

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

Problem: Learn how to calcuate line integrals.

Directions: The group works as a team, each person doing a different part of the question. Help each other out, explain what you are doing.

1. Goal: Evaluate the line integral $\int_C x+y \, ds$ where C is the path given by $x(t) = 2t^2$, $y(t) = 1-3t^2$, $t \in [0,1]$.

r': $x' = 4t$ $y' = -6t$

Person 1: Calculate $ds = |r'(t)|dt = \sqrt{16t^2 + 36t^2}\, dt$

$= \sqrt{52t^2}\, dt$

$= \sqrt{52}\, t\, dt$

Person 2: Substitute $x(t), y(t), ds$ into the integral

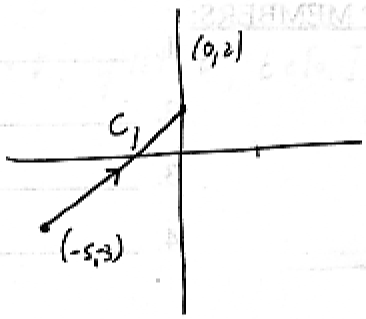
$$\int_0^1 (\underbrace{2t^2}_{x} + \underbrace{(1-3t^2)}_{y})\sqrt{52}\, t \, dt$$

Person 3: Evaluate the integral

$$\int_0^1 (t - t^2)\sqrt{52}\, dt = \sqrt{52}\left(\frac{t^2}{2} - \frac{t^3}{3}\right)\Big|_0^1 = \sqrt{52}\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{\sqrt{52}}{6}$$

2. Goal: Evaluate the line integral $\int y^2\,dx + x\,dy$ where C_1 is the line segment from $(-5,-3)$ to $(0,2)$.



Person 4: Find the parametric equations $x(t)$, $y(t)$ for this line. Make $(x(0) = -5, y(0) = -3)$, $(x(1) = 0, y(1) = 2)$.

$$l(t) = p_t + t\vec{v}$$

$$= (-5,-3) + t(5,5) = r(t) = (\underbrace{-5+5t}_{x(t)}, \underbrace{-3+5t}_{y(t)})$$

$$r(0) = (-5,-3) \qquad r(1) = (0,2)$$

Person 1: Calculate dx and dy in terms of dt.

$$dx = x'\,dt = 5\,dt \qquad dy = y'\,dt = 5\,dt$$

Person 2: Substitute $x(t), y(t), dx, dy$ into the integral.

$$\int_0^1 (-3+5t)^2\,5\,dt + (-5+5t)\,5\,dt$$

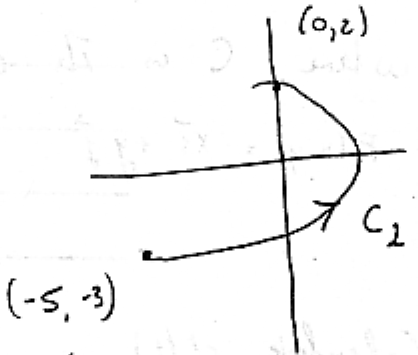
$$\int_0^1 (9 - 30t + 25t^2)5 - 25 + 25t \quad dt$$

$$= \int_0^1 20 - 125t + 125t^2\,dt = 20t - \frac{125t^2}{2} + \frac{125}{3}t^3\Big|_0^1 = 20 - \frac{125}{2} + \frac{125}{3}$$

Person 3: Evaluate the integral.

$$= -\frac{5}{6}$$

3. Goal: Evaluate the line integral $\int_{C_2} y^2\,dx + x\,dy$ where C_2 is the parabola $x = 4 - y^2$, $y \in [-3, 2]$.



<u>Person 4</u>: Calculate dx for this parametrization.

$$dx = -2y\,dy$$

<u>Person 1</u>: Substitute $x(y), y, dx$ into the integral to get an integral in terms of y and dy.

$$\int_{-3}^{2} y^2(-2y\,dy) + (4-y^2)\,dy$$

$$= \int_{-3}^{2} -2y^3 + 4 - y^2 \;dy$$

<u>Person 2</u>: Evaluate the integral.

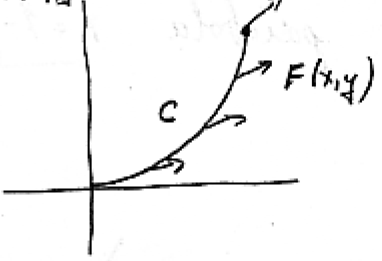
$$= -\frac{2y^4}{2} + 4y - \frac{y^3}{3}\Big|_{-3}^{2} = 40.83$$

<u>Person 3</u>: Compare the values of (2) and (3). Note that C_1, C_2 begin and end at the same points.

$$\int_{C_1} = -5/6 \neq 40.83 = \int_{C_2}$$

4. Goal: Evaluate the line integral of the vector field $\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{F} \cdot \vec{T}\, ds = \int_C \vec{F} \cdot r'\, d$

where C is the curve $x(t) = t$, $y(t) = t^2$, $t \in [0,1]$,
and $F(x,y) = x\vec{i} + y\vec{j}$.



Person 1: Calculate $r'(t)$.

Person 2: Evaluate $\vec{F}(x,y) \cdot \vec{r}\,'(t)$

Person 3: Substitute all values into the integral.

Person 4: Evaluate the integral.