Polar Coordinates Integral

GROUP MEMBERS:

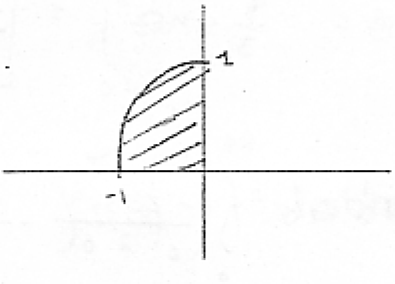
1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________

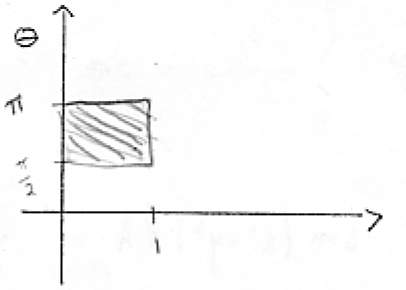
Problem: Learn how to calcuate integrals using polar coordinates.

Directions: One person works at a time . Explain to the group what you are doing. Write out your work and answer on the sheet. When done, ask whether everyone in the group understands. People in the group ask questions. Pass the sheet to person 2 who will follow the same procedure. Then person 3 and so on.

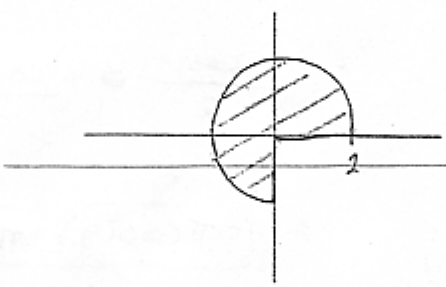
For each of the following regions, describe the regions in terms of r and θ. Draw the regions in the (r, θ) plane.

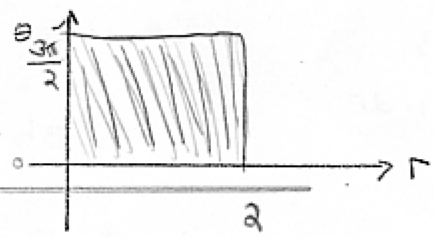
1.





2.





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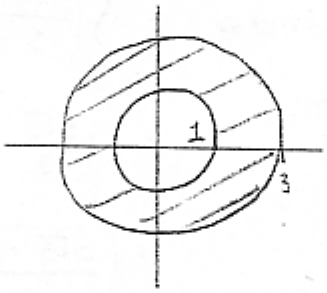

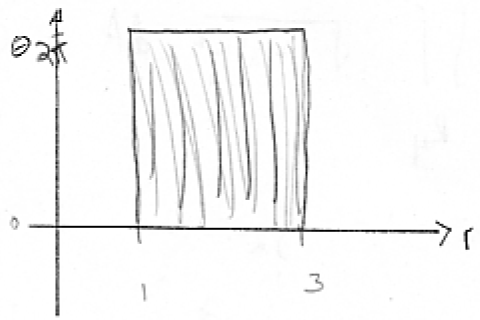

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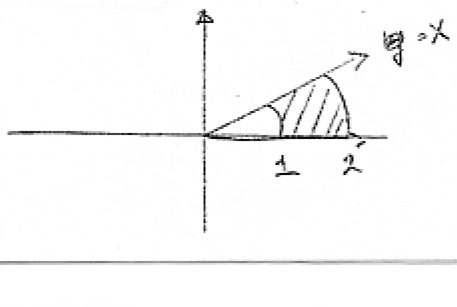

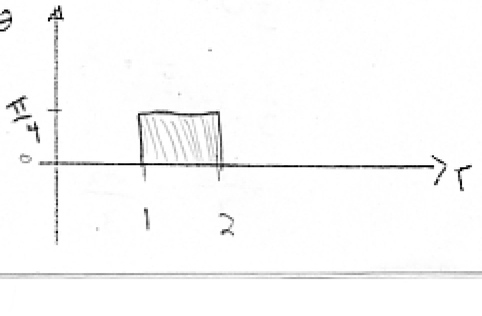

For each of the regions from the previous page, transform the integral by making the polar coordinates change of variables. (Don't integrate yet).

1. $\iint_{R_1} x^2+y^2 \, dA = \int_{\frac{\pi}{2}}^{\pi}\int_0^1 r^2 \, r\,dr\,d\theta = \int_{\frac{\pi}{2}}^{\pi}\int_0^1 r^3 \, dr\,d\theta$

$\int_{\frac{\pi}{2}}^{\pi}\int_0^1 r^3 \, dr\,d\theta = \int_{\frac{\pi}{2}}^{\pi} \frac{r^4}{4} d\theta \Big|_0^1 = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{4} d\theta = \frac{1}{4}\theta \Big|_{\frac{\pi}{2}}^{\pi} = \frac{\pi}{8}$

2. $\iint_{R_2} x \, dA = \int_0^{\frac{3\pi}{2}}\int_0^2 r\cos\theta \, r\,dr\,d\theta = \int_0^{\frac{3\pi}{2}}\int_0^2 r^2\cos\theta \, dr\,d\theta$

$\int_0^{\frac{3\pi}{2}}\int_0^2 r^2\cos\theta \, dr\,d\theta = \int_0^{\frac{3\pi}{2}} \frac{r^3}{3}\cos\theta \, d\theta \Big|_0^2 = \int_0^{\frac{3\pi}{2}} \frac{8}{3}\cos\theta \, d\theta = \frac{8}{3}\sin\theta \Big|_0^{\frac{3\pi}{2}} = \boxed{\frac{-8}{3}}$

3. $\iint_{R_3} \sin(x^2+y^2) \, dA = \int_0^{2\pi}\int_1^3 \sin(r^2)\, r\,dr\,d\theta = \int_0^{2\pi}\left(\frac{-\cos(9)}{2} + \frac{\cos(1)}{2}\right) d\theta$

$u = r^2$

$du = 2r\,dr$

$\frac{du}{2} = r\,dr$

$= \frac{-\cos(9)}{2}\theta + \frac{\cos(1)}{2}\theta \Big|_0^{2\pi}$

$= \boxed{-\pi\cos(9) + \pi\cos(1)}$

$\int_1^3 \sin(r^2)\, r\,dr$

$= \int_1^3 \frac{1}{2}\sin u \, du$

$= \frac{1}{2}(-\cos u) \Big|_1^3 = \frac{1}{2}(-\cos(r^2)) \Big|_1^3 = \frac{-\cos(9)}{2} + \frac{\cos(1)}{2}$

4. $\iint_{R_4} \sqrt{1+x^2+y^2} \, dA = \int_0^{\frac{\pi}{4}}\int_1^2 \sqrt{1+r^2}\, r\,dr\,d\theta = \int_0^{\frac{\pi}{4}}\left(\frac{5\sqrt{5}}{3} - \frac{2\sqrt{2}}{3}\right) d\theta$

$u = 1+r^2$

$du = 2r\,dr$

$\frac{du}{2} = r\,dr$

$= \left(\frac{5\sqrt{5}}{3} - \frac{2\sqrt{2}}{3}\right)\theta \Big|_0^{\frac{\pi}{4}}$

$= \boxed{\left(\frac{5\sqrt{5} - 2\sqrt{2}}{3}\right)\frac{\pi}{4}}$

$\int_1^2 \sqrt{1+r^2}\, r\,dr = \int_1^2 \frac{1}{2}u^{\frac{1}{2}}\,dr$

$= \frac{1}{3}u^{\frac{3}{2}} \Big|_1^2$

$= \frac{1}{3}(1+r^2)^{\frac{3}{2}} \Big|_1^2$

$= \frac{5\sqrt{5}}{3} - \frac{2\sqrt{2}}{3}$

Everyone together perform the following integration:

$$\int_0^1\int_0^{2\pi} d\theta\, r\,dr = \int_0^1 \theta\, r\,dr \Big|_0^{2\pi}$$

$$= \int_0^1 2\pi r\,dr$$

$$= \pi r^2 \Big|_0^1$$

$$= \pi$$

$$\int_0^{\pi}\int_0^{1} r^2\,dr\,d\theta = \int_0^{\pi} \frac{r^3}{3}\,d\theta \Big|_0^1$$

$$= \int_0^{\pi} \frac{1}{3}\,d\theta$$

$$= \frac{1}{3}\theta \Big|_0^{\pi}$$

$$= \frac{\pi}{3}$$