

Polar Coordinates Integral

GROUP MEMBERS:

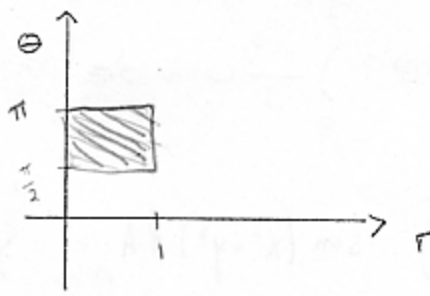
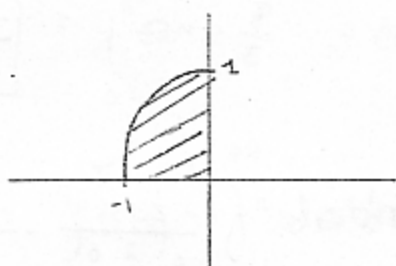
1. _____
2. _____
3. _____
4. _____

Problem: Learn how to calculate integrals using polar coordinates.

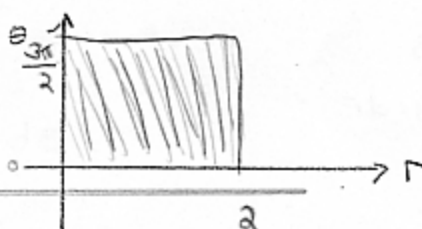
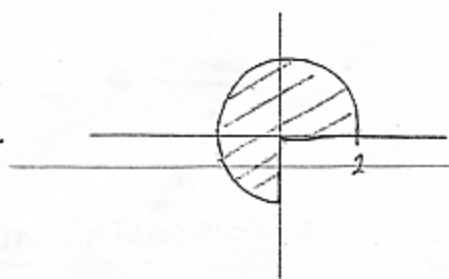
Directions: One person works at a time. Explain to the group what you are doing. Write out your work and answer on the sheet. When done, ask whether everyone in the group understands. People in the group ask questions. Pass the sheet to person 2 who will follow the same procedure. Then person 3 and so on.

For each of the following regions, describe the regions in terms of r and θ . Draw the regions in the (r, θ) plane.

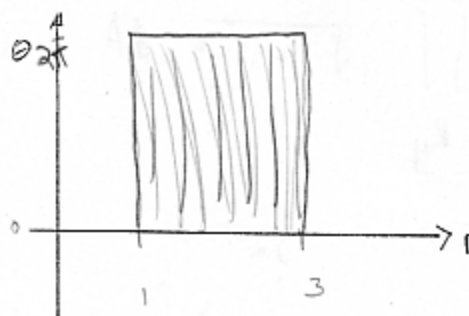
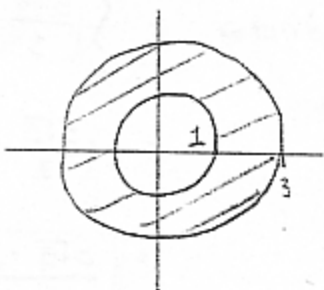
1.



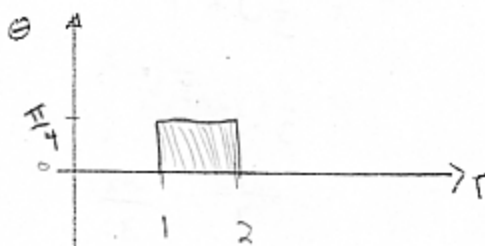
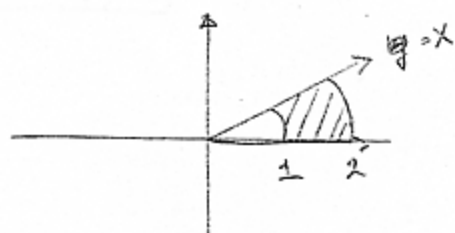
2.



3.



4.



For each of the regions from the previous page, transform the integral by making the polar coordinates change of variables. (Don't integrate yet).

$$1. \iint_{R_1} x^2 + y^2 \, dA = \int_{\frac{\pi}{2}}^{\pi} \int_0^1 r^2 \, r \, dr \, d\theta = \int_{\frac{\pi}{2}}^{\pi} \int_0^1 r^3 \, dr \, d\theta$$

$$\int_{\frac{\pi}{2}}^{\pi} \int_0^1 r^3 \, dr \, d\theta = \int_{\frac{\pi}{2}}^{\pi} \left. \frac{r^4}{4} \right|_0^1 d\theta = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{4} d\theta = \frac{1}{4} \theta \Big|_{\frac{\pi}{2}}^{\pi} = \frac{\pi}{8}$$

$$2. \iint_{R_2} x \, dA = \int_0^{\frac{3\pi}{2}} \int_0^2 r \cos \theta \, r \, dr \, d\theta = \int_0^{\frac{3\pi}{2}} \int_0^2 r^2 \cos \theta \, dr \, d\theta$$

$$\int_0^{\frac{3\pi}{2}} \int_0^2 r^2 \cos \theta \, dr \, d\theta = \int_0^{\frac{3\pi}{2}} \left. \frac{r^3}{3} \cos \theta \right|_0^2 d\theta = \int_0^{\frac{3\pi}{2}} \frac{8}{3} \cos \theta \, d\theta = \frac{8}{3} \sin \theta \Big|_0^{\frac{3\pi}{2}} = \boxed{\frac{-8}{3}}$$

$$3. \iint_{R_3} \sin(x^2 + y^2) \, dA = \int_0^{2\pi} \int_1^3 \sin(r^2) \, r \, dr \, d\theta = \int_0^{2\pi} \left(\frac{-\cos(\theta)}{2} + \frac{\cos(1)}{2} \right) d\theta$$

$$\int_1^3 \sin(r^2) \, r \, dr$$

$u = r^2$
 $du = 2r \, dr$
 $\frac{du}{2} = r \, dr$

$$= \int_0^{2\pi} \left(\frac{-\cos(\theta)}{2} + \frac{\cos(1)}{2} \right) d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \sin u \, du$$

$$= \left. \frac{1}{2} (-\cos u) \right|_1^3 = \frac{1}{2} (-\cos(r^2)) \Big|_1^3 = \frac{-\cos(9)}{2} + \frac{\cos(1)}{2}$$

$$= \boxed{-\pi \cos(9) + \pi \cos(1)}$$

$$4. \iint_{R_4} \sqrt{1+x^2+y^2} \, dA = \int_0^{\frac{\pi}{4}} \int_1^2 \sqrt{1+r^2} \, r \, dr \, d\theta = \int_0^{\frac{\pi}{4}} \left(\frac{5\sqrt{5}}{3} - \frac{2\sqrt{2}}{3} \right) d\theta$$

$$\int_1^2 \sqrt{1+r^2} \, r \, dr$$

$u = 1+r^2$
 $du = 2r \, dr$
 $\frac{du}{2} = r \, dr$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{5\sqrt{5}}{3} - \frac{2\sqrt{2}}{3} \right) d\theta$$

$$\int_1^2 \sqrt{1+r^2} \, r \, dr = \int_1^2 \frac{1}{2} u^{\frac{1}{2}} \, du$$

$$= \frac{1}{3} u^{\frac{3}{2}} \Big|_1^2$$

$$= \frac{1}{3} (1+r^2)^{\frac{3}{2}} \Big|_1^2$$

$$= \frac{5\sqrt{5}}{3} - \frac{2\sqrt{2}}{3}$$

$$= \boxed{\left(\frac{5\sqrt{5} - 2\sqrt{2}}{3} \right) \frac{\pi}{4}}$$

Everyone together perform the following integration:

$$\int_0^1 \int_0^{2\pi} r \, dr \, d\theta = \int_0^1 \left[\theta r \, dr \right]_0^{2\pi} \\ = \int_0^1 2\pi r \, dr$$

Explain: Learn how to calculate integrals over polar coordinates.

Explain: Get person wears a tire = πr^2 | what you're looking. What
can you work and answer at the sheet. What is the order appropriate to the group
rule-governed. People in the group ask questions of the sheet to person who will
follow the same procedure. They refer to the sheet.

For each of the following integrals, describe the region in terms of r and θ . Draw the
regions in the (r, θ) plane.

$$\int_0^{\pi} \int_0^1 r^2 \, dr \, d\theta = \int_0^{\pi} \left[\frac{r^3}{3} \, d\theta \right]_0^1 \\ = \int_0^{\pi} \frac{1}{3} \, d\theta \\ = \frac{1}{3} \theta \Big|_0^{\pi} \\ = \frac{\pi}{3}$$