Triple Integrals

GROUP MEMBERS:

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________

Problem 1: Set up the limits of integration for a triple integral $\iiint_R f(x,y,z)\,dV$ where

$R = \{(x,y,z) \mid 0 \leq x \leq 2,\ 1 \leq y \leq 4-x,\ 0 \leq z \leq 16-x^2-y^2\}$.

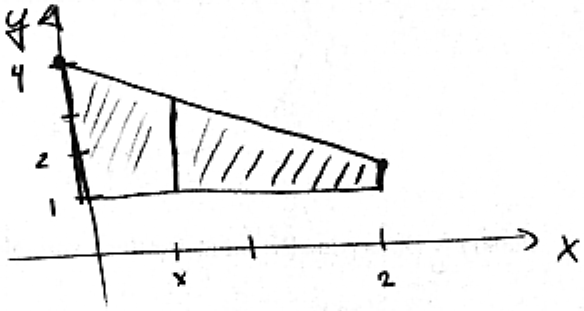
Fill in the limits of integration for the integral and put the dx, dy, dz in the correct order:

$$\int_0^2 \int_1^{4-x} \int_0^{16-x^2-y^2} f(x,y,z) \qquad dz\,dy\,dx$$

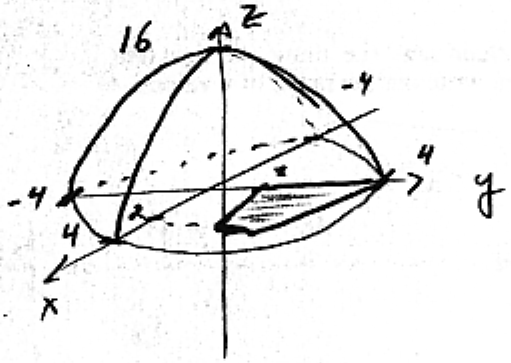
Person 1: Put the limits for the outer integral.
Person 2: Put the limits for the middle integral.
Person 3: Put the limits for the inner integral.

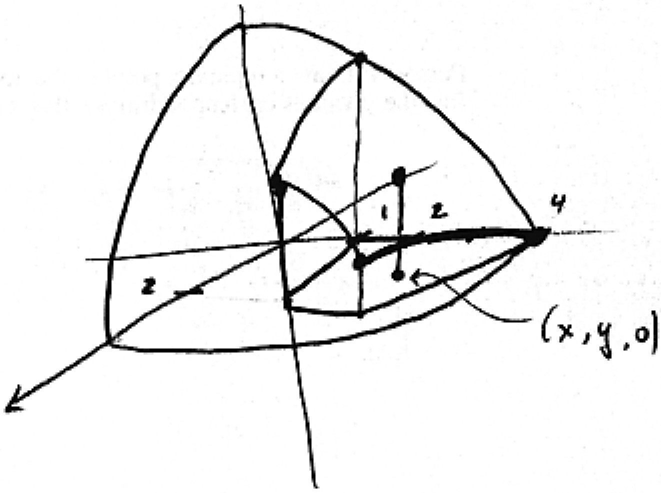
Problem 2:

Person 4: Make a xy-plane and in this plane draw the part of R given by $\{(x,y,z) \mid 0 \leq x \leq 2,\ 1 \leq y \leq 4-x,\ z = 0\}$.



Group: Make a sketch in xyz space showing the surface $z = 16 - x^2 - y^2$ with $0 \leq z$. Then in this 3 dimension picture sketch the region from the xy plane that was drawn above. From this picture, try to visualize the region R.

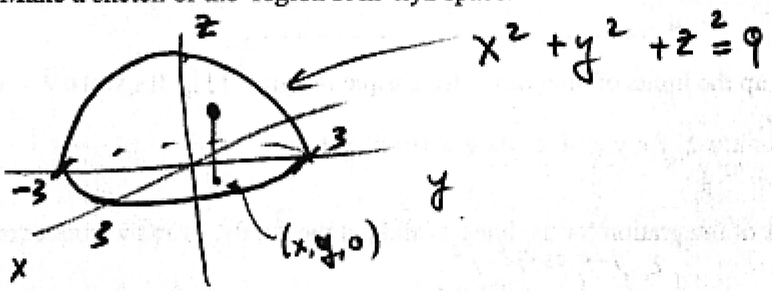




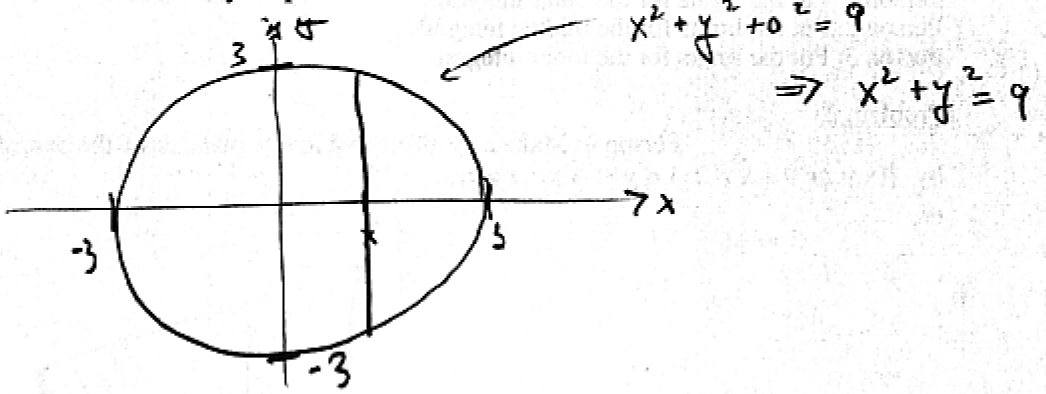
<u>Problem 3:</u> Set up the limits of integration for a triple integral

$\int\int\int_R x^2+y^2+z^2 \; dV$ where E is the region inside the sphere $x^2+y^2+z^2=9$ and above the xy plane.

Person 1: Make a sketch of the region R in xyz space.



Person 2: Make a sketch on the xy plane showing the projection of the region R onto the xy plane. Label the curves in your picture.



Person 3: Determine the limits for the x variable using the above sketch.

$x \in [-3, 3]$

Person 4: Draw a typical x point in the above sketch, "chain saw" (i.e. draw a vertical line for) the y values corresponding to this x point and then write out the range of y values.

$-\sqrt{9-x^2} \le y \le +\sqrt{9-x^2}$

Person 1: In the sketch of the region R (on the previous page), put in a typical point (x,y, 0) and then "chain saw" the z values above it (i.e. Draw a vertical line in the z direction showing the z values). Write out the range of the z values.

$$0 \le z \le \sqrt{9-x^2-y^2}$$

Person 2: Using the above limits, set up the endpoints for the triple integral

$\iiint_R x^2+y^2+z^2 \ dV$ (Do not evaluate)

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{+\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} (x^2+y^2+z^2)\,dz\,dy\,dx$$

Problem4: Convert the integral $\iiint_R x^2+y^2+z^2\, dV$ into spherical coordinates.

Person 3: Give the limits of the ρ variable.

$0 \le \rho \le 3$

Person 4: Give the limits of the ϕ variable.

$0 \le \phi \le \pi/2$

Person 1: Give the limits of the ϑ variable.

$0 \le \theta \le 2\pi$

Person 2: Convert dV = dx dy dz into spherical coordinates.

$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

Person 3: Convert the function $f(x,y,z) = x^2+y^2+z^2$ into spherical coordinates.

$= \rho^2$

Person 4: Set up the endpoints for the integral in spherical coordinates.

$$\int_0^3 \int_0^{\pi/2} \int_0^{2\pi} (\rho^2) \quad \rho^2 \sin\phi \, d\theta \, d\phi \, d\rho$$

Person 1: Write out the integral completely converted into spherical coordinates.

Everybody: evaluate the integral.

$$\int_0^3 \int_0^{\pi/2} \rho^2 \sin\phi \left(\int_0^{2\pi} d\theta\right) d\phi \, d\rho = \int_0^3 \int_0^{\pi/2} \rho^2 \sin\phi \cdot 2\pi \, d\phi \, d\rho$$

$$= \int_0^3 2\pi\rho^2 \left(\int_0^{\pi/2} \sin\phi \, d\phi\right) d\rho = \int_0^3 2\pi\rho^2 \cdot 1 \, d\rho = \frac{2\pi\rho^3}{3}\Big|_0^3$$

$$\int_0^{\pi/2} \sin\phi \, d\phi = -\cos\phi \Big|_0^{\pi/2} = -\cos\pi/2 + \cos 0 = 1$$

$$= 18\pi$$