

Linear Approximation

GROUP MEMBERS:

1. _____
2. _____
3. _____
4. _____

Problem: Calculate partial derivatives and linear approximation from a contour graph. Use the attached contour plot. Assume the value of $f(x,y)$ represents the temperature in degrees Celsius and the units on the xy plane are km. The origin is the Park Science Building.

Directions: Person 1 take this sheet and do problem 1. Explain to the group what you are doing. Ask your group for help if you are not clear about what to do. Then pass the sheet to person 2 who does the second problem and explains to the group. Then pass to the third person and so on.

A. Estimate $\frac{\partial f}{\partial x}$

Person 1. Plot the point (2,1).

Person 2: Estimate $f(2,1)$. Include the units.

Person 3: Choose a new point near to (2, 1) that has the same y value but a different x value. Call this new point $(x_{new}, 1)$. What is the coordinate of your new point? What is the f value at the new point?

Person 4: Estimate $\frac{\partial f}{\partial x}$ at $(x=2, y=1)$ by taking the difference quotient

$$\frac{\partial f}{\partial x} \approx \frac{f(x_{new},1) - f(2,1)}{x_{new} - 2}.$$

What are the units for $\frac{\partial f}{\partial x}$?

B. Estimate $\frac{\partial f}{\partial y}$

Person 1: Choose a new point near to (2, 1) that has the same x value but a different y value. Call this new point (2, y_{new}). What is the coordinate of your new point? What is the f value at the new point?

Person 2: Estimate $\frac{\partial f}{\partial y}$ at (x=2, y=1) by taking the difference quotient

$$\frac{\partial f}{\partial y} \approx \frac{f(2, y_{new}) - f(2, 1)}{y_{new} - 1}.$$

What are the units for $\frac{\partial f}{\partial y}$?

B. Use the **linear approximation** for f to estimate the value of f near (2,1). The linear approximation, also called the **linearization**, is the same as the formula for the tangent plane approximation.

$$z = L(x, y) = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(y - y_0)$$

Person 3: Using the values for the partial derivatives calculated above, write out the formula for the linear approximation based at (x₀=2, y₀=1).

L(x,y) =

Person 4: Use this linear approximation formula to estimate the value of the temperature at the point (x = 2.03, y = 1.02).

Everyone: Use this linear approximation formula to estimate the value of the temperature at the point (x = 1.98, y = .97).

Tangent Line Approximation:

Problem: Determine the tangent vector and tangent line to a curve given by

$$\mathbf{r}(t) = (x(t), y(t), z(t)) = (\cos(t), \sin(t), t)$$

at the point $\mathbf{r}(t = \pi/2)$.

Person 1: Find the point of interest: $\mathbf{r}(\pi/2) =$

Person 2: Calculate the tangent vector $\mathbf{r}'(t) =$

Person 3: Calculate the tangent vector at $t = \pi/2$: $\mathbf{r}'(\pi/2) =$

Person 4: Write the equation of the tangent line through the point $\mathbf{r}(\pi/2)$.