Triple Integrals

GROUP MEMBERS:

1. _____
2. _____
3. _____
4. _____

Problem 1: Set up the limits of integration for a triple integral $\iiint_R f(x,y,z)\, dV$ where

$R = \{(x,y,z) \mid 0 \leq x \leq 2,\ 1 \leq y \leq 4-x,\ 0 \leq z \leq 16 - x^2 - y^2\}$.

Fill in the limits of integration for the integral and put the dx, dy, dz in the correct order:

$$\int \quad \int \quad \int \qquad f(x,y,z)$$

Person 1: Put the limits for the outer integral.
Person 2: Put the limits for the middle integral.
Person 3: Put the limits for the inner integral.

Problem 2:

Person 4: Make a xy-plane and in this plane draw the part of R given by $\{(x,y,z) \mid 0 \leq x \leq 2,\ 1 \leq y \leq 4-x,\ z = 0\}$.

Group: Make a sketch in xyz space showing the surface $z = 16 - x^2 - y^2$ with $0 \leq z$. Then in this 3 dimension picture sketch the region from the xy plane that was drawn above. From this picture, try to visualize the region R.

Problem 3: Set up the limits of integration for a triple integral

$\iiint_R x^2+y^2+z^2 \; dV$ where E is the region inside the sphere $x^2+y^2+z^2 = 9$ and above the xy plane.

Person 1: Make a sketch of the region R in xyz space.

Person 2: Make a sketch on the xy plane showing the projection of the region R onto the xy plane. Label the curves in your picture.

Person 3: Determine the limits for the x variable using the above sketch.

Person 4: Draw a typical x point in the above sketch, "chain saw" (i.e. draw a vertical line for) the y values corresponding to this x point and then write out the range of y values.

Person 1: In the sketch of the region R (on the previous page), put in a typical point (x,y, 0) and then "chain saw" the z values above it (i.e. Draw a vertical line in the z direction showing the z values). Write out the range of the z values.

Person 2: Using the above limits, set up the endpoints for the triple integral

$\iiint_R x^2+y^2+z^2 \; dV$ (Do not evaluate)

Problem4: Convert the integral $\iiint_R x^2+y^2+z^2 \; dV$ into spherical coordinates.

Person 3: Give the limits of the ρ variable.

Person 4: Give the limits of the ϕ variable.

Person 1: Give the limits of the ϑ variable.

Person 2: Convert dV = dx dy dz into spherical coordinates.

Person 3: Convert the function $f(x,y,z) = x^2+y^2+z^2$ into spherical coordinates.

Person 4: Set up the endpoints for the integral in spherical coordinates.

Person 1: Write out the integral completely converted into spherical coordinates.

Everybody: evaluate the integral.