Path Integrals

Group Members:

1.
2.
3.
4.

Evaluate the path integral \( \int_{C} \vec{F} \cdot \vec{T} \, ds \) where \( C \) is the path given by the parametrization

\[
\vec{r}(t) = (x(t) = t^2, y(t) = -t^3), \ t \in [0, 1]
\]

and the vector field is given by

\[
\vec{F}(x, y) = (x^2y^3, -y\sqrt{x}).
\]
1. For \( f(x, y) = x^2y^3 \) calculate \( \nabla f = (\partial_x, \partial_y) f = (\partial_x f, \partial_y f) \).

2. What is the relationship between \( \nabla f \) and \( F(x, y) = (P(x, y) = 2xy^3, Q(x, y) = 3x^2y^2) \)?

3. When \( F(x, y) = \nabla f \) then the Fundamental Theorem for Line Integrals says that

\[
\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} = f(B) - f(A)
\]

where the path \( C \) starts at the point \( A \) and finishes at the point \( B \).

Use this theorem to evaluate the path integral \( \int_C \vec{F} \cdot d\vec{r} \) for

\[
F(x, y) = (P(x, y) = 2xy^3, Q(x, y) = 3x^2y^2)
\]

where \( C \) is any path that starts at \( A = (0, 0) \) and finishes at \( B = (2, 1) \).
Person 1: Evaluate the path integral
\[ \int_{C_1} \vec{F} \cdot \vec{T} \, ds = \int_{C_1} \vec{F} \cdot d\vec{r} \]

where \( C_1 \) is the path given by the parametrization
\[ \vec{r}(t) = (x(t) = 2t, y(t) = t^2), \ t \in [0, 1] \]

and the vector field is given by
\[ F(x, y) = (xy, y^3). \]
1. For \( f(x, y) = x^2y^3 \) calculate \( \nabla f = (\partial_x, \partial_y) f = (\partial_x f, \partial_y f) \).

2. What is the relationship between \( \nabla f \) and \( F(x, y) = (P(x, y) = 2xy^3, Q(x, y) = 3x^2y^2) \).

3. When \( F(x, y) = \nabla f \) then the Fundamental Theorem for Line Integrals says that

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Person 2: Evaluate the path integral

\[ \int_{C_2} \vec{F} \cdot \vec{T} \, ds = \int_{C_1} \vec{F} \cdot d\vec{r} \]

where \( C_2 \) is the path given by the parametrization

\[ \vec{r}(t) = (x(t) = 2t, y(t) = t), \quad t \in [0, 1] \]

and the vector field is given by

\[ F(x, y) = (xy, y^3). \]
1. For \( f(x, y) = x^2y^3 \) calculate \( \nabla f = \nabla (\partial_x, \partial_y)f = (\partial_x f, \partial_y f) \).

2. What is the relationship between \( \text{grad} f \) and \( F(x, y) = (P(x, y) = 2xy^3, Q(x, y) = 3x^2y^2) \).

3. When \( F(x, y) = \nabla f \) then the Fundamental Theorem for Line Integrals says that

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where \( C \) is any path that starts at \( A = (0, 0) \) and finishes at \( B = (2, 1) \).
Person 3: Evaluate the path integral
\[
\int_{C_1} \vec{F} \cdot \vec{T} \, ds = \int_{C_1} \vec{F} \cdot d\vec{r}
\]
where \(C_1\) is the path given by the parametrization
\[
\vec{r}(t) = (x(t) = 2t, y(t) = t^2), \quad t \in [0, 1]
\]
and the vector field is given by
\[
F(x, y) = (2xy^3, 3x^2y^2).
\]
1. For \( f(x, y) = x^2y^3 \) calculate grad \( f = \nabla f = (\partial_x, \partial_y)f = (\partial_x f, \partial_y f) \).

2. What is the relationship between \( \text{grad } f \) and \( F(x, y) = (P(x, y) = 2xy^3, Q(x, y) = 3x^2y^2) \).

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where \( C \) is any path that starts at \( A = (0, 0) \) and finishes at \( B = (2, 1) \).
Person 4: Evaluate the path integral

\[ \int_{C_2} \vec{F} \cdot \vec{T} \, ds = \int_{C_1} \vec{F} \cdot d\vec{r} \]

where \( C_2 \) is the path given by the parametrization

\[ \vec{r}(t) = (x(t) = 2t, y(t) = t), \quad t \in [0, 1] \]

and the vector field is given by

\[ F(x, y) = (2xy^3, 3x^2y^2). \]
1. For \( f(x, y) = x^2y^3 \) calculate \( \text{grad } f = \nabla f = (\partial_x, \partial_y)f = (\partial_x f, \partial_y f) \).

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