Homework on Linearization:

Sect 5.1 # 1, 8

Consider the non-linear system that we examined in class:

\[
\frac{dx}{dt} = (x - y)(1 - x - y)
\]
\[
\frac{dy}{dt} = x(2 + y)
\]

a. Find the equilibrium points for this system. (Hopefully you will find the same points we found in class: (0, 0), (0, 1), (–2, 2), (3, –2). ) Show your calculations clearly. On the midterm you will have to do this type of calculation so be sure you understand how to find the equilibrium points.

b. Calculate the linearized system at each of the equilibrium points. At the point (-2,2), use the change of variables method. At all the other points, use whichever method you find easier (change of variables or Jacobian matrix).

c. For each equilibrium point, calculate the eigenvalues and, when the eigenvalues are real, also calculate the eigenvectors. Do two of the equilibrium points by hand; you can use Mathematica for the other two if you like. Using the eigenvalues and eigenvectors, draw the phase picture in the neighborhood of each equilibrium point. Make 4 separate pictures.

d. Make a phase picture for the whole space by combining your 4 separate pictures.