Let $A: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a transformation given by the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Claim: The transformation is linear so that for every constant $k_1$ and $k_2$ and every vector $v_1 = (x_1, y_1)$ and $v_2 = (x_2, y_2)$ we have that:

$$A(k_1 v_1 + k_2 v_2) = k_1 A(v_1) + k_2 A(v_2).$$

Proof:

Step 1. Prove that $A(k_1 v_1) = k_1 A(v_1)$

Step 2: Prove that $A(v_1 + v_2) = A(v_1) + A(v_2)$.

Step 3: Use Step 1 and Step 2 to show that the claim is true.