Wk 3 Homework: due Wed Feb 13.

1. Prove that if $P_n = (x_n, y_n)$ converges to $P = (x, y) \in (R^2, d_E)$ then $x_n \to x$ and $y_n \to y$.

2. a. Show that the taxi cab metric $d_{TC}$ (which is defined as $d_1$ in Example 11.4.2, Bartle) is a metric.
   
b. Draw the unit ball in the taxi cab metric. This is problem #8a, p. 333.

3. Prove: If a set $S$ is closed then every limit point of $S$ is contained in $S$. Before starting this problem, list the different strategies you might use to help figure out what to do when you do not know what to do.

4. Consider the half-infinite interval $\{x \in R : x \geq 0\}$. This set together with the absolute value distance defines a metric space. Is this metric space complete?

5. a. Give an example of an open set $G$ in $R$ such that
   
a. $G$ is bounded.
   
b. $G$ is unbounded.
   
c. $G$ is connected (i.e. one piece).
   
d. $G$ is disconnected (has two or more pieces).
   
b. Repeat (a) but now for $G$ a closed set.

6. Consider the space $C[0, 1]$. Let $f(x) = \frac{2}{3}x + 1$ and $g(x) = x^2$ be elements of the function space. Calculate $d_{\infty}(f, g)$ and $d_1(f, g)$. (Hint: How do you find the maximum value of a function?).

7. (Sect 8.1) For the sequences of functions $f_n(x) = x^2/n$, $x \in [0, 1]$. (a) make an animation of the sequence (b) calculate the pointwise limit of the sequence of functions.

From Bartle: p. 333 #6, 9; p. 232 #1, 3.