Hw 9: Power Series
Sect 9.4:
p. 272:
5 (Hint: apply ratio test).
6ace

17: Do the following steps for this problem.
a. Find a power series that equals \( f(x) = \frac{1}{1+x^2} \). Determine the radius of convergence \( R \) for this power series.
b. Use Theorem 8.2.4 to justify the interchange of integral and limit:

\[
\int f(x) \, dx = \int \left( \lim_{n \to \infty} s_n(x) \right) \, dx = \lim_{n \to \infty} \int s_n(x) \, dx
\]

and thereby get a power series expression for arctan \( x \) that holds for \( |x| \leq r \) where \( r \) is any number less than \( R \) =radius of convergence found in part a.
c. Explain why this gives that for all \( x \) satisfying \( |x| < 1 \),

\[
\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1} x^{2n+1}
\]

Sect 8.2, p. 238

#1, 2,

10: Set \( f_n(x) = e^{-nx} \), for \( x \in [1, 2] \). Determine the pointwise limit \( f_n(x) \to f(x) \) for \( x \in [1, 2] \). Then prove that this convergence is uniform for \( x \in [1, 2] \). Finally use the integration theorem).