

"Reform" Calculus

A Presentation for **ED 220:**
**Changing Pedagogies in Math
and Science Education**

Rob Manning
Math Department
Haverford College
Feb. 26, 2004

Traditional Calculus

- Focus on algebraic manipulations
- Conceptual material tends to be abstract and intense (e.g., epsilon-delta definitions of limit, proof of mean value theorem)
- Some “applied” problems, but most are pseudo-real-world (e.g., related rates, geometric applications of integrals only)

Sample Integration Problems (Traditional)

Evaluate the following integrals:

$$\frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$\frac{1+x}{1+x^2} dx$$

$$\int_0^1 (1-x)^9 dx$$

$$\int_0^1 \frac{e^x}{1+e^x} dx$$

(Integration problems on Math 111 Final Exam, Cornell, Fall 1993)

“Reform” Calculus

- Focus away from algebraic manipulations (less on calculating limits, derivatives, integrals), perhaps replaced by Mathematica, integral tables, etc.
- Emphasize *concepts* of limit, derivative, integration via conceptual HW problems
- Connect to more applications (though many can still feel fake)

Sample Integration Problem (Reform)

The circumference of a tree at different heights above the ground is given in the table below. Assume that all horizontal cross-sections of the tree are circles. Estimate the volume of the tree.

Height (in)	0	20	40	60	80	100	120
Circumference (in)	31	28	21	17	12	8	2

The “Rule of Three”

Give equal weight to 3 perspectives on a function:

- Graphical (a picture)
- Numerical (a table of values)
- Algebraic (a formula)

Example (integration):

- Given graph of velocity, answer questions about position (antiderivative of velocity)
- Given tabular data, compute integral (tree)
- Given formula, compute integral (traditional)

Writing (and the “Rule of Four”)

- In addition to previous three perspectives, also have students express math in their own words
- Standard “numerical answer” problems too easily done by mimicking textbook examples
- Requiring explanations in good English often reveals misunderstanding that would be hidden in standard HW papers.
- Fight against bad habits developed in high school (and college) of treating math as a sequence of equations rather than logical arguments.

Sample Integration Problem (Writing)

Goal: Make 2 million candies in 200 days

Student Tasks (see handout)

- Develop integral (and explain in writing) for production and storage cost for a given production schedule
- Evaluate cost for uniform schedule algebraically
- Invent smart non-uniform schedule (more candies earlier reduces storage cost); evaluate its cost numerically
- Revise cost model to better reflect reality

Strengths of Calculus Reform

- Focuses more on ideas of calculus, less on rote calculations – students have to think when doing HW
- Writing projects/applied problems more representative of how math is used in the real world
- Takes advantage of new technologies (calculator, computer algebra) to shift emphasis to more interesting topics

Weaknesses of Calculus Reform

- Students are bad at algebra: this only makes things worse!
- Some classic topics used in later courses may be jettisoned (e.g., trig substitution, partial fractions, hyperbolic functions, some series convergence tests)

References

Traditional texts (?):

- Stewart, Calculus (Wadsworth)
- Anton, Calculus (Wiley)
- Finney/Thomas, Calculus (Pearson)

Reform text:

- Hughes-Hallett et al = “(non)-Harvard calculus”

Writing in Math:

- Meier/Rishel, Writing and Teaching of Mathematics (MAA)
- Cohen et al, Students Research Projects in Calculus (MAA)