**Bifurcations**

**Goal:** To study the effect of various levels of harvesting on the steady state fish population where the population is assumed to grow logistically.

\[
\frac{dP}{dt} = P(1 - P) - h = -P^2 + P - h
\]

The value of \( h \) denotes the number of fish caught per year. When \( h = 0 \), no fish are harvested and the model reduces to the standard logistic model

\[
\frac{dP}{dt} = kP(1 - \frac{P}{N})
\]

where we are assuming \( k = 1 \) and \( N = 1 \).

**Directions:** Examine the equation for different values of \( h \). Draw the phase line that corresponds to each value of \( h \) and label the equilibrium points. The group will break up and find people who are working with the same \( h \) values.

Person 1: \( h = 0 \), \( h = 0.6 \)
Person 2: \( h = 0.05 \), \( h = 0.5 \)
Person 3: \( h = \frac{1}{8} \), \( h = 0.4 \)
Person 4: \( h = 0.2 \), \( h = 0.3 \)

When you return to your group, you will combine all your results into one picture. For each \( h \) value, you will plot the phase line for that \( h \) value as a vertical line in the \((h, y)\) plane. The union of all these phase line pictures will help us understand how the fish populations vary as we vary the harvesting amount \( h \).

Go in order of increasing \( h \) values. One at a time, each person adds their phase line picture to the overall picture. Explain what equilibrium points you calculated and describe their type.
Group Questions:

1. “Connect the dots” that represent the equilibrium values so you have a (rough) estimate of the equilibrium values for all \( h \) values, rather than just the few you have calculated exactly.

2. What happens to the fish population over the long term if the fishing level \( h \) is high?

3. As fishermen, we want to be able to both catch lots of fish to earn money but at the same time make sure that we do not over fish and kill off the fish population. What is the highest sustainable level of yearly fish catch (i.e. the fishing level) that the system can support? Give your answer with units. Assume that \( P(t) \) has units of millions of fish. Let time \( t \) be given in units of year.

4. As you think of varying the fishing level \( h \), is there a value that acts as a dividing line between a sustainable fish population and a fish population that will die out? What is this value of \( h \)?