Calculus Review and Introduction to Differential Equations

Due Thursday, Jan 21 at the start of class. You will be going over your answers with your classmates as you work in groups.

In all your work, write out your answers clearly, showing the steps. Your goal is to convince a skeptical reader that your answer is correct.

1. Review of differentiation:
   • Give the derivatives of the basic calculus functions: \( x^n, \cos x, \sin x, \tan x, e^x, \ln x \).
   • Write out the basic rules of differentiation (power rule, product rule, quotient rule, chain rule)
   • Illustrate each rule with an example.

2. Review of Integration:
   • Give the integrals of the basic calculus functions: \( x^n, 1/x, \cos x, \sin x, e^x \).
   • Describe the basic techniques of integration (substitution, integration by parts) and illustrate each with an example.

3. Guess and Check for a simple differential equation:
   • For the function \( y(t) = e^{4t} \), calculate \( y'(t) \). Does the function satisfy the differential equation \( y'(t) + 2y(t) = 0 \)? Does the function satisfy the differential equation \( y'(t) - 4y(t) = 0 \)? Does the function satisfy the differential equation \( y'(t) = 4y(t) \)?
   • For the function \( y(t) = e^{-7t} \), calculate \( y'(t) \). Does the function satisfy the differential equation \( y'(t) - 5y(t) = 0 \)? Does the function satisfy the differential equation \( y'(t) + 7y(t) = 0 \)? Does the function satisfy the differential equation \( y'(t) = -7y(t) \)?
   • For the function \( y(t) = e^{2t} \), calculate \( y'(t) \). Write down a differential equation that this function solves. Be sure to show that the function satisfies the equation you have written.

4. Think ahead
   • Generalization: For \( y(t) = e^{kt} \) where \( k \) stands for a real number, what differential equation does \( y(t) \) satisfy?
   • Does the function \( y(t) = 7e^{4t} \) satisfy \( y'(t) = 4y(t) \)? Does the function \( y(t) = -3e^{4t} \) satisfy \( y'(t) = 4y(t) \)? Are there other functions that satisfy \( y'(t) = 4y(t) \)?