Complex Eigenvalues Worksheet

Consider the linear system

\[
\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -3 & -5 \\ 3 & 1 \end{pmatrix} \mathbf{Y}
\]

1. Find the eigenvalues, \( \lambda_1 \) and \( \lambda_2 \).

2. Is the origin a spiral sink, spiral source, or center?

3. Determine the natural period of the oscillations.

4. Determine the direction of the oscillations in the phase plane (do solutions travel clockwise or counterclockwise around the origin)?

5. Find the general solution using the following steps:
   a. Find an eigenvector, \( \mathbf{v} \), for one of the eigenvalues.
b. Write the corresponding complex solution.

c. Rewrite this using Euler’s formula (write in the form \( Y(t) = Y_{re}(t) + iY_{im}(t) \)).

d. Recall that \( Y_{re}(t) \) and \( Y_{im}(t) \) are both real solutions of the linear system \( \frac{dY}{dt} = AY \). Write the general (real) solution, \( Y(t) \).

6. Find the particular solution with the initial condition \( Y_0 = (4,0) \).

7. Sketch the \( x(t) \)- and \( y(t) \)-graphs of the particular solution.