Partial Derivative Applications

1. **Crawling Speed of Larvae**: In a study on the Northern Blowfly, the crawling speed of the larvae was found to be a function of body length and ambient temperature [1],

\[ v(l, T) = 5.45 \log l + 0.66T - 12.8, \]

where \( v \) is the speed (cm/min) of the larvae, \( l \) is the length of the larvae, and \( T \) is the ambient temperature. Notice an increase in body length or an increase in temperature will increase the crawling speed. This result is very useful in forensic entomology (the study of arthropod biology to criminal matters) for estimating the time that elapsed following the larva’s departure from the corpse [1]. Find \( dv/dl \) and \( dv/dT \).

2. **Spruce Budworm**: A parasitoid is an organism that attaches to or within a host during part of their development. Unlike parasites, parasitoids ultimately kill their hosts. The Nicholson-Bailey model is a frequently used model to describe the population dynamics of the host-parasitoid system, in which it is assumed that the number of parasitized hosts, denoted by \( N_a \), is given by

\[ N_a = N[1 - e^{-bP}], \]

where \( N \) is the host density, \( P \) is the parasitoid density, and \( b \) is the searching efficiency of the parasitoid.

(a) Show that

\[ b = \frac{1}{P} \log \frac{N}{N - N_a}, \]

by solving for \( b \) in the equation above.

(b) Consider \( b \) as a function of \( P, N, N_a \):

\[ b = f(P, N, N_a) = \frac{1}{P} \log \frac{N}{N - N_a}. \]

How is the searching efficiency \( b \) affected when the parasitoid density increases? [2]

**References**
