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Science problems in math courses:

How does computer modeling fit into a math/science curriculum?

Rob Manning
Mathematics Department
Haverford College

Many problems in introductory math texts are:

- short ($\leq$ 30 minutes of student work, half-page chain of equations leading to an answer)
- artificial (solve a given equation, maybe with interpretation given)
- not well connected to material covered in science courses

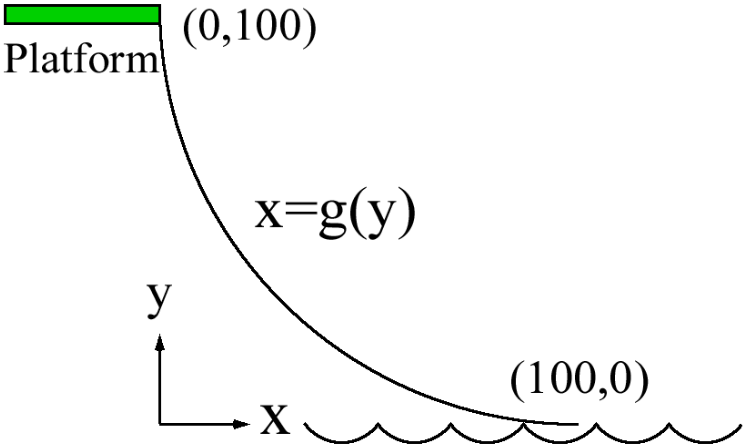
Alternatives: “projects”, “labs”, “papers”

I have tried incorporating such longer problems into several courses:

- **10-page papers in calculus**
- biweekly computer labs in differential eqns
- biweekly computer labs in sci computing

The Ultimate Water Slide

Themes: Building a definite integral, improper integrals, numerical integration on calculator.



You are a consultant in the design of a new water slide. Customers will take an elevator to a platform 100 ft above a large pool, grab a sled, and glide down a metal slide to the pool below. The end of the slide is to be at a horizontal distance of 100 ft from the platform.

This project investigates what slide shape will make the total travel time t_{ride} as short as possible.

(a) For a given slide shape $x = g(y)$, find a definite integral over y for t_{ride} in terms of $g(y)$, using the principle of conservation of energy. At $(0, 100)$, the rider (mass m) has total energy $100mg$ (zero kinetic energy, 100mg gravitational potential energy). At some (x, y) on the slide, the rider has mgy potential energy and kinetic energy $mv^2/2$. Determine the speed v of the rider at (x, y). Slice the slide into small pieces, and use your formula for v to determine the time to travel on a typical little piece, and from this derive the definite integral over y for t_{ride}.

(b) Consider a slide with shape $y = 100(1 - \sqrt{\frac{x}{100}})$. Sketch the slide. Solve for x to find $g(y)$. Determine t_{ride} for this slide using numerical integration on your calculator (or Mathematica). The integral will be improper, so you may have to take some special care in order to get a numerical answer from your calculator.

(c) Show that for $p \geq 1$, $\int_0^{100} \frac{1}{(100-y)^p} dy$ diverges, while for $0 \geq p < 1$, $\int_0^{100} \frac{1}{(100-y)^p} dy$ converges.

(d) Repeat (b) for $y = 100(1 - (\frac{x}{100})^p)$ with $p = \frac{1}{4}, 1, 2, 4$. Before going to your calculator in each case, figure out whether t_{ride} converges by looking carefully at the integrand and deciding which member of the family in (c) it behaves like as $y \to 100$. For those that converge, compute t_{ride} on your calculator. For those that diverge, explain physically why that makes sense. Make a table of t_{ride} as a function of p.

(e) Find the value of p between 0 and 2 that minimizes t_{ride}. You will not be able to determine this exactly, but experiment numerically to determine p to within 0.01. Sketch the shape of the slide at this p value. Why do you think this shape beats all other p values?

(f) Discuss any way of making this problem more realistic. For example, the current model assumes no friction or air resistance, so you could discuss how you would begin to add one of them in some reasonable way.

The Happy Pumpkin Candy Factor

Themes: Building a definite integral, numerical integration on calculator.

As a consultant to the Happy Pumpkin Candy Factory, you must design the production schedule for their popular chocolate-covered-aspargus Halloween candies. It is now 200 days before Halloween, and the factory must have 2 million candies produced by Halloween. Past research shows that to produce candies at a rate of R candies per day costs the factory $50 + \frac{R}{1000}$ cents per candy. To store A candies costs $\frac{A}{2}$ cents per day (to rent storage space, power the refrigerators, hire security guards, etc.)

(a) Let t be the time (in days) from today, and let $C(t)$ be the number of candies produced by time t (thus, the total output from today to time t). Find a definite integral over t for the total production cost from now until Halloween in terms of $C(t)$. Slice the time interval $[0, 200]$ into pieces; figure out the cost to the factory from time t to time $t + dt$.

(b) Consider a uniform production schedule (produce candies at a constant rate so that you just produce 2 million candies by Halloween). Determine the total production cost for this schedule.

(c) Design a production schedule that has a smaller cost than the uniform schedule in (b). Try to minimize the total production cost as best you can. You will need to experiment a bit; please report the results for several of your experiments, and some intuitive reasoning for why your schedules cost less than the uniform schedule. Make sure you produce exactly 2 million candies by Halloween in each case.

With the current model for storage, the cost of storing A candies for one day is a linear function $\frac{A}{2}$. A more realistic model for storage costs would be that the cost is an increasing but concave down function $f(A)$.

(d) Why might such a function f be a more realistic model of storage costs?

(e) Find a function $f(A)$ that is increasing and concave down for $A > 0$, which is approximately equal to $\frac{A}{2}$ for A small, but starts to deviate somewhat from $\frac{A}{2}$ when A is around 1 million. Make a plot of $f(A)$ and $\frac{A}{2}$ together.

(f) Repeat (a)–(c) for the model with $f(A)$ replacing $\frac{A}{2}$ as the storage cost. Depending on your function $f(A)$, you may need to compute integrals numerically. Discuss differences in results for this new storage cost model.

(g) Discuss another way in which this model is unrealistic. Adapt the model to respond to your objection, and repeat (a)–(c) for your new model.

Why use such assignments?

- As with any word problem, develops modeling skills: turn problem description in words into mathematical notation

- Gives experience with open-ended problems

- Students experience a variety of approaches to problem-solving: intuition combined with traditional computation and technology use

- **Students must write; then you really see what they understand**

- Connects with students' interests and experience with other science courses

Challenges in using such assignments

- Hard to find problems that are “realistic” but doable (really, they will at best be “semi-realistic”)

- Want to make sure your approach to another field is not in conflict with what students see in that other field (I’m neither an economist nor a computational chemist)

- More work (design of good projects, meeting with student groups, grading drafts and final papers)