Physics 122 Exam #1
Spring 2015

Solutions

You are subject to the Bryn Mawr College honor code while taking this exam. This is a closed book exam. You may use a simple calculator and a pencil during this exam, but nothing else. Do all questions on the exam. Do all work directly on the examination, in the space provided. If you run out of room, you may use the back of the page. Be sure to answer all parts of each question. Some problems will take longer than others, but all will be weighted equally when graded. When providing a sketch, please be as neat, accurate, and quantitative as possible. A sloppy sketch indicates misunderstanding and will receive a reduced grade. The final page of this exam provide several potentially useful equations. You may remove this page for easy reference.

This exam will be available between Friday, 27 February 2015 and Monday, 2 March 2015, at the reserve desk in the Collier Science Library during normal library operating hours. I will collect all of the exams Tuesday morning when the library opens. Take the exam during a single 90 minute sitting in the library.

Before you begin, write your name in the space below and at the top of EVERY page of this exam, note the date in the space provided on this page, and note the time. When you finish, note the time and return the exam to the circulation desk. The difference between your start and finish times should be no more that 90 minutes.

Name:

Date:

Start time:

Finish time:
1. For the swinging pendulum shown below, (a) make a diagram indicating
the acceleration of the bob at positions \( P \) (the end of the swing) and
\( Q \) (the bottom of the swing), and (b) draw force diagrams of the bob
when it is at positions \( P \) and \( Q \). In each force diagram, show both the
individual forces as well as the net (resultant) force, if any, acting on
the bob. (As a more familiar example think of a child on a playground
swing.)

![Diagram of swinging pendulum](image)

The speed is momentarily zero at point \( P \) so there is no radial acceleration. However,
there is a force acting tangentially, which changes the speed at this point (the bob
doesn't remain stopped).

The speed is not zero at point \( Q \) so there
must be radial component of acceleration.
However, no forces act tangentially so there
is no tangential acceleration.
2. A ball is thrown at an angle $\theta$ up to the top of a cliff of height $L$, from a distance $L$ from the base as shown below. Find an expression for the speed necessary to make the ball hit right at the edge of the cliff. Your answer should only include the quantities given, $L$ and $\theta$ and known constants such as $g$.

\[ x = x_0 + v_{0x} t \]
\[ y = y_0 + v_{0y} t + \frac{1}{2} a t^2 \]
\[ L = 0 + v_{0x} \sin \theta \cdot t - \frac{1}{2} g t^2 \]
\[ t = \frac{L}{v_{0x} \cos \theta} - \frac{1}{2} g \frac{L}{v_{0x}^2 \cos^2 \theta} \]
\[ 1 = \tan \theta - \frac{1}{2} g \frac{L}{v_{0x}^2 \cos^2 \theta} \]
\[ v_{0x}^2 = \frac{1}{2} g \frac{L}{\cos^2 \theta (\tan \theta - 1)} \]
\[ v_{0x} = \frac{L}{\cos \theta} \sqrt{\frac{1}{2} \left( \frac{g L}{\sin \theta \cos \theta (\tan \theta - 1)} \right)} \]
3. Two stones are released from rest at a certain height, one after the other. (a) Will the difference in their speeds increase, decrease, or stay the same? (b) Will their separation distance increase, decrease, or stay the same? (c) Will the time interval between the instants at which they hit the ground be smaller than, equal to, or larger than the time interval between the instants of their release? Explain your reasoning in each case.

(a) The difference in their speeds will stay the same as they fall. The difference in speeds is shown by the vertical lines between the two straight lines in the v-t curves. Alternatively, we could use our constant acceleration kinematic equations: let stone 1 be released at time zero and stone 2 at time T. Then the velocity of each at some later time is \( v_1 = -gt \) and \( v_2 = -g(t-T) \). Their difference is \( v_2 - v_1 = gT \), which is independent of time.

(b) Their separation increases as seen by the vertical lines in the x-t curves. Or using kinematics, \( x_1 = -\frac{1}{2}gt^2 \) and \( x_2 = -\frac{1}{2}g(t-T)^2 \). So \( x_2 - x_1 = gT + \frac{1}{2}gT^2 \), which increases with time t.

(c) The time interval will be the same as seen in the x-t curve above.
4. An object of mass $m_1$ attached to a spring of force constant $k$ slides on a frictionless horizontal surface. It oscillates with an amplitude $A$. When the spring is at its greatest extension and the mass, $m_1$, is instantaneously at rest, a second object of mass $m_2$ is placed on top of the original mass. What is the smallest value that the coefficient of static friction can have if the second object is not to slip on the first? Explain how the amplitude, angular frequency and period of the oscillating system are changed by placing $m_2$ on $m_1$.

\[ F_{sys} = (m_1+m_2)a \]
\[ kA = (m_1+m_2)a \]
\[ f = \frac{m_2a}{m_2} \]
\[ \mu = \frac{kA}{(m_1+m_2)g} \]

The amplitude $A$ doesn't change. 

The angular freq $\omega = \sqrt{\frac{k}{M}}$ before. $\omega = \sqrt{\frac{k}{m_1}}$, after $\omega = \sqrt{\frac{k}{m_1+m_2}}$. 

The frequency is less with $m_2$. 

The period $T = \frac{2\pi}{\omega}$, period is more with $m_2$. 

\[ f_s \rightarrow [m_1] \rightarrow f \]
\[ f_s \rightarrow [m_1] \rightarrow f \]
\[ f_s \rightarrow [m_2] \rightarrow f \]
Potentially useful equations

\[ v_x = v_{0x} + a_x t \]
\[ x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \]
\[ v_x^2 = v_{0x}^2 + 2a_x (x - x_0) \]

\[ r = r\dot{r} \]
\[ v = r\dot{r} + r\ddot{\theta} \]
\[ a = (\ddot{r} - r\dot{\theta}^2)\dot{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\dot{\theta} \]

\[ a_c = \frac{v^2}{r} = \omega^2 r \]
\[ v = r\omega \]
\[ \omega = 2\pi f = \frac{2\pi}{T} \]

\[ F = ma \]
\[ F_{\text{friction}} = \mu N \]
\[ W = mg \]
\[ F_s = -kx \]
\[ F_{\text{grav}} = \frac{GM_aM_b}{r^2} \]

for \( \ddot{q} + \omega^2 q = 0 \), \( q(t) = A \sin(\omega t + \phi) \)

\[ \mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta \]
\[ \mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z \]
\[ |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}| |\mathbf{B}| \sin \theta \]
\[ \mathbf{A} \times \mathbf{B} = i(A_y B_z - A_z B_y) - j(A_x B_z - A_z B_x) + k(A_x B_y - A_y B_x) \]

\[ g = 10 \text{ m/s}^2 \]
\[ G = 6.673 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2} \]

for \( ax^2 + bx + c = 0 \), \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)