A small block of mass \( m \) starts from rest and slides along a frictionless loop-the-loop as shown in the sketch on the next page. What should be the initial height \( z \), so that \( m \) pushes against the top of the track (at \( a \)) with a force equal to its weight?

At the top the net force is in the central direction

\[
F = ma_c
\]

\[
mg + N = \frac{mv^2}{R}
\]

The problem tells us that \( N = mg \)

So

\[
mg + mg = \frac{mv^2}{R}
\]

\[
2gR = v^2
\]

\[
v = \sqrt{2gR}
\]

Now use energy conservation to find release height

\[
mgz = mg(2R) + \frac{1}{2}mv^2
\]

\[
Z = 2R + \frac{1}{2} \times \frac{v^2}{g} = 2gR
\]

\[
Z = 3R
\]
5.3 Ballistic pendulum

A simple way to measure the speed of a bullet is with a ballistic pendulum. As illustrated, this consists of a wooden block of mass \( M \) into which the bullet is shot. The block is suspended from cables of length \( l \), and the impact of the bullet causes it to swing through a maximum angle \( \phi \), as shown. The initial speed of the bullet is \( v \), and its mass is \( m \).

(a) How fast is the block moving immediately after the bullet comes to rest? (Assume that this happens quickly.)

(b) Show how to find the velocity of the bullet by measuring \( m, M, l, \) and \( \phi \).

\[
\begin{align*}
a) \text{using momentum conservation} \\
mv &= (m+M)v' \\
v' &= \frac{m}{m+M}v \\
b) \text{now we can use energy conservation} \\
\frac{1}{2} (m+M) v'^2 &= (m+M)gh \\
\cos \phi &= \frac{e-h}{l} \\
h &= l - l \cos \phi \\
\text{putting these all together} \\
\frac{1}{2} v'^2 &= gh \\
\frac{1}{2} \left( \frac{m}{m+M} \right)^2 v^2 &= g(l - l \cos \phi) \\
\left( \frac{m}{m+M} \right)^2 v^2 &= 2g(l - l \cos \phi) \\
v &= \left( \frac{m+M}{m} \right) \sqrt{2gL(1-\cos \phi)}
\end{align*}
\]
5.4 Sliding on a circular path

A small cube of mass \( m \) slides down a circular path of radius \( R \) cut into a large block of mass \( M \), as shown. \( M \) rests on a table, and both blocks move without friction. The blocks are initially at rest, and \( m \) starts from the top of the path.

Find the velocity \( v \) of the cube as it leaves the block.

**Conservation of Energy**

\[
mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2
\]

grav P.E.
change of \( m \)

K.E. of \( m \)
when it reaches the bottom

K.E. of \( M \)
where \( m \)
is at the bottom

**Momentum Conservation**

\[
0 = mv - MV
\]

combine these

\[
mgR = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{m}{M}v\right)^2
\]

\[
mgR = \frac{1}{2}mv^2 + \frac{1}{2}\frac{m^2}{M}v^2
\]

\[
2gR = v^2\left(1 + \frac{m}{M}\right)
\]

\[
v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}
\]

If \( m = M \)

\[
v = \sqrt{\frac{2gR}{2}} = \sqrt{gR}
\]
5.6 Block sliding on a sphere*

A small block slides from rest from the top of a frictionless sphere of radius $R$, as shown on the next page. How far below the top $x$ does it lose contact with the sphere? The sphere does not move.

When the block loses contact with the sphere, the normal force from the sphere on the block will be zero.

Using Newton's 2nd law for the central direction:

$$ F = ma $$

$$ mg \cos \theta = m \frac{v^2}{R} $$

Now find $v$ using energy conservation:

$$ mgx = \frac{1}{2}mv^2 $$

Relate $\cos \theta$ to $x$:

$$ \cos \theta = \frac{R-x}{R} $$

Combining those three:

$$ mg\left(\frac{R-x}{R}\right) = \frac{2mgx}{R} $$

$$ R-x = \frac{2x}{R} $$

$$ R-x = 2\frac{x}{3} $$

$$ x = \frac{R}{3} $$