2. Basic Q ideas that emerged from SGz:
   i) Quantization of measured variable:
      they measured $S_z$, found only $\pm \hbar/2$
   ii) Apparently, cannot know some pairs of
       variables (e.g. $S_z$ and $S_x$) simultaneously
   iii) Q systems exhibit interference effects, which
       demand treatment in terms of amplitudes.

   **Analogy w/polarized light**
   Expt 2: $\uparrow \rightarrow \downarrow$
   $\rightarrow \uparrow$
   $\rightarrow \downarrow$

   **Explanation for the light version:**
   $\text{transmission axis}$
   $\text{we choose } E_{in} = 1 \text{ N/C}$
   $\Rightarrow E_{out} = \cos \theta (N/C)$
   $= \hat{\text{out}} \cdot \hat{\text{in}} (N/C)$

   $\Rightarrow$ the amplitude of transmission depends both on
   the incident light & on the property required for
   the light to get through.

   $\Rightarrow$ Similarly, for a quantum exp., the amplitude
   depends both on the initial state and the state
   being detected.
We associate with each ket, e.g. $|a\rangle$, a corresponding "bra" $\langle a|$.

By analogy with $E_{\text{out}} = \hat{E}_{\text{out}} \cdot \hat{E}_{\text{in}}$ (N/C), we define the "inner product" $\langle b|a\rangle$ to be the amplitude of detection for an electron initially in the state $|a\rangle$ which we're trying to detect in the state $|b\rangle$. The probability of detection is $|\langle b|a\rangle|^2$.

Brief review of complex numbers

$$z = a + ib = |z| \cos \Theta + i |z| \sin \Theta$$
$$|z| = \sqrt{a^2 + b^2}$$

Euler: $e^{i\Theta} = \cos \Theta + i \sin \Theta \Rightarrow z = |z|e^{i\Theta}$

Form the complex conjugate by $i \mapsto -i$:
$z^* = a - ib = |z|e^{-i\Theta}$
$|z|^2 = |z|^2 e^{0} = |z|^2$
$= (a+ib)(a-ib) = a^2 + b^2 = |z|^2$

What do we already know about probability amplitudes?

Expt 1:

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>+2\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>-2\rangle$</td>
</tr>
</tbody>
</table>

$|\langle +2|+2\rangle|^2 = 1 \Rightarrow \langle +2|+2\rangle = e^{i\Theta}$

Prob. ampl. for electron in $|+2\rangle$ to be detected in $|+2\rangle$ amplitude: we'll show that it has no physical consequence $\Rightarrow$ choose $\Theta = 0$

$\Rightarrow \langle +2|+2\rangle = 1$

Expt 1 also tells us that

$|\langle -2|-2\rangle|^2 = 0 \Rightarrow \langle -2|-2\rangle = 0$

Prob. ampl. for electron in $|+2\rangle$ to be detected in $|-2\rangle$.

Similarly, an alternate version of the expt:

$|\langle +2|-2\rangle|^2 = 0 \Rightarrow \langle +2|-2\rangle = 0$

Expt 3:

$|\langle +2|+x\rangle|^2 = \frac{1}{2} \Rightarrow \langle +2|+x\rangle = \frac{e^{i\delta} + \sqrt{2}}{\sqrt{2}}$