Magnet resonance (cont.)

\[ |4(t)\rangle \rightarrow |a(t)\rangle \rightarrow \text{in the absence of photons} \quad b = b_0 e^{i\omega t/2} \]

Factor out this very first time dependence:

\[ a = c(t) e^{-i\omega t/2} \quad b = d(t) e^{i\omega t/2} \]

\[ \rightarrow \begin{pmatrix} \dot{c} \\ \dot{d} \end{pmatrix} = -\frac{i\omega}{4} \begin{pmatrix} 1 & e^{i(\omega - \omega_0) t} + e^{-i(\omega - \omega_0) t} \\ e^{i(\omega_0 - \omega) t} + e^{-i(\omega_0 + \omega) t} \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} \]

When \(\omega = \omega_0\)

\[ h\omega_0 = \text{difference in energy levels due to } B = B_0 \hat{k} \]

\[ \begin{pmatrix} \dot{c} \\ \dot{d} \end{pmatrix} = -\frac{i\omega}{4} \begin{pmatrix} e^{2i\omega t} + 1 \\ 1 + e^{-2i\omega t} \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} \]

For timescales \(\gg \frac{1}{\omega_0}\), the \(e^{2i\omega t}\) & \(e^{-2i\omega t}\) terms average to zero, leaving the 1.

\[ \rightarrow |4(t)\rangle \rightarrow \begin{pmatrix} \cos \frac{\omega t}{2} e^{-i\omega t/2} \\ -i \sin \frac{\omega t}{2} e^{i\omega t/2} \end{pmatrix} \]

\( t\)-basis

\[\rightarrow P_\pm = \frac{1}{2} \left( 1 - \cos \frac{\omega t}{2} \right) \]

P prob to be in 1-2\rangle

\[ P_+ = \frac{1}{2} \left( 1 + \cos \frac{\omega t}{2} \right) \]

In the presence of photons, can get "stimulated emission," i.e. photon-induced transition from high-energy state to low-energy state, with emission of a photon.

Or, can absorb a photon & promote from lower energy state to higher energy state.

Note: the \(B_0\) perceived by a particular proton in a molecule depends on the shielding effect & magnetic field of other nearby electrons.

\[\Rightarrow\] If \(B_0\) is fixed and \(\omega\) of photon is varied, multiple resonance peaks (\(\omega = \omega_0\)) are found, corresponding to the different microenvironments of the protons: NMR, MRI
Let's see what happens if we initially localize the system in configuration $1\rangle$:

$$|\psi(0)\rangle = 1\rangle = \frac{1}{\sqrt{2}} \left( |1\rangle + |3\rangle \right)$$

$$\rightarrow |\psi(t)\rangle = \frac{e^{-i(E_0 - A)t/\hbar}}{\sqrt{2}} \left( |1\rangle + e^{-2iAt/\hbar} |3\rangle \right)$$

We'll use $|1\rangle$ and $|3\rangle$ (the configurations shown above) as our basis states, even though they're not quite orthogonal.

In this basis, $\hat{A} \rightarrow \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}$

$$\rightarrow E = E_0 \pm A$$

$E = E_0 - A \leftrightarrow |1\rangle \rightarrow \frac{1}{\sqrt{2}} (|1\rangle - |3\rangle)$

$E = E_0 + A \leftrightarrow |3\rangle \rightarrow \frac{1}{\sqrt{2}} (|1\rangle + |3\rangle)$