Ammonia (ctd.)

Better explanation for why $A \propto \frac{1}{\text{coupling strength}}$:

1. $|2\rangle$ is almost an energy eigenstate
2. $\hat{A}|2\rangle \approx E_0|2\rangle$
3. $-A = \langle 1|\hat{A}|2\rangle \approx E_0 \langle 1|2\rangle$

If we start the system in $|1\rangle$, which is an equal superposition of the energy eigenstates, then it oscillates to $|2\rangle$, back to $|1\rangle$, etc. with period $\frac{2\pi}{A}$.

The same thing happens for any double-well potential.

This is analogous to the behavior of a coupled oscillator:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>left bob oscillates</td>
<td>right bob still</td>
<td>left bob still</td>
<td>right bob oscillates</td>
</tr>
</tbody>
</table>

Energy-time uncertainty relation

Townsend shows $\Delta E \Delta t \geq \hbar/2$, where

$\Delta t \equiv \text{time for the state to change significantly}$.

But, can also interpret $\Delta t$ as the time used to measure $E$, and $\Delta E$ as the measurement uncertainty $\equiv$ can violate cons. of energy, so long as you're quick about it!

Time evolution of expectation values for energy eigenstates

$\langle A \rangle_t = \langle A \rangle_{t=0}$

For any observable $\hat{A} \equiv$ "nothing ever changes in an energy eigenstate."

Connecting (again) to the wavefunction

$|\Psi\rangle = \sum C_n |E_n\rangle$

$C_n = \langle E_n | \Psi \rangle$

$\hat{I} = \sum a_i \hat{X} a_i^\dagger$

any complete basis

$|\psi(x)\rangle = \int |x\rangle \langle x| \Psi \rangle dx$

$C_n = \int \langle E_n | x \rangle \langle x | \Psi \rangle dx = \int \Psi^*_n \psi(x) dx$
Two distinguishable spin-$\frac{1}{2}$ particles
(e.g. electron & proton)

Our basis for describing this system:

$|1\rangle \equiv |+z\rangle \otimes |+z\rangle$

state of state of
particle 1 particle 2

$|2\rangle \equiv |-z\rangle \otimes |-z\rangle$

$|3\rangle \equiv |+z\rangle \otimes |-z\rangle$

$|4\rangle \equiv |-z\rangle \otimes |+z\rangle$

⇒ an arbitrary state $|4\rangle$ would be written

$|4\rangle \rightarrow |1\rangle \langle 1| + |2\rangle \langle 2| + |3\rangle \langle 3| + |4\rangle \langle 4|$

The direct product

$|+z\rangle \otimes |-z\rangle$

two-particle state

$|+z\rangle$ one-particle state for

particle 1

$|-z\rangle$ one-particle state for

particle 2

to express as a matrix:

$|+z\rangle \otimes |-z\rangle$

$A \otimes B$

first write

this in matrix form

then multiply each
element by the matrix form
of this

\[ \hat{S}_{12} = \hat{S}_{1} \otimes \hat{I} \rightarrow \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & -\frac{i}{2} \end{pmatrix} \]

example: $\hat{S}_{12} |1\rangle \rightarrow \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix} |1\rangle = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$

In general, if $\hat{C} = \hat{A} \otimes \hat{B}$, then you'll show on your next assignment that

$\hat{C} \rightarrow \begin{pmatrix} |1\rangle \langle 1| & |1\rangle \langle 2| & |1\rangle \langle 3| & \cdots \\ |2\rangle \langle 1| & |2\rangle \langle 2| & |2\rangle \langle 3| & \cdots \\ |3\rangle \langle 1| & |3\rangle \langle 2| & |3\rangle \langle 3| & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

\[ = \begin{pmatrix} |+z\rangle \langle +z| + |-z\rangle \langle +z| + |z\rangle \langle +z| & \cdots & \cdots \\ |+z\rangle \langle +z| + |-z\rangle \langle +z| + |z\rangle \langle +z| & \cdots & \cdots \\ \cdots & \cdots & \cdots & \ddots \end{pmatrix} \]