Scattering & Tunneling (cont.)

\[ E > V_0 \]

\[ \rightarrow \text{To left of step } \Psi = Ae^{ikx} + Be^{-ikx} \quad k = \sqrt{2m(E-V_0)}/\hbar \]

\[ \text{To right of step, } \Psi = Ce^{i\alpha x} \quad Q = \sqrt{2m(E-V_0)}/\hbar \]

B & C can be determined in terms of A using continuity of \( \Psi \) & \( d\Psi/dx \) at \( x=0 \).

\[ \rightarrow \text{For a right-propagating wave } \Psi = Ae^{ikx}, \quad j = 1A^2 \]

\[ \text{For a left-propagating wave } \Psi = Be^{-ikx}, \quad j = -1B^2 \]

For \( E < V_0 \) (classically, particle is forbidden from \( x>0 \))

\[ \rightarrow \text{To left of step: Same as above} \]

\[ \rightarrow \text{To right of step, } \Psi = Ce^{-ikx} + De^{ikx} \quad \text{where } k = \sqrt{2m(V_0-E)}/\hbar, \text{ "evanescent wave"} \]

not a propagating wave

\[ \rightarrow j = 0 \text{ for } x>0, \text{ as expected.} \]

Used continuity of \( \Psi \) & \( d\Psi/dx \) at \( x=0 \) to find B and C in terms of A, using Mathematica.

Tunneling

\[ \begin{align*}
    x=0 & : \quad \Psi = Ae^{ikx} + Be^{-ikx} \\
    0<x<a & : \quad \Psi = Ce^{-kx} + De^{kx} \quad \text{(cut & discard, since this solution is only valid for } x \neq a) \\
    x>a & : \quad \Psi = Fe^{ikx} \\
\end{align*} \]

Apply continuity of \( \Psi \) & \( d\Psi/dx \) at \( x=0 \) & \( x=a \) to find B, C, D, & E in terms of A (using Mathematica).

\[ \rightarrow \text{For rare tunneling (i.e., for } k\alpha \gg 1), \]

the probability of transmission is

\[ T = \frac{\text{trans}}{\text{incident}} \approx \frac{(4k\alpha)^2}{(k^2 + \hbar^2)^2} e \]

Scanning Tunneling Microscopy (STM)

Tip > Surface

Work function \( \phi \) is the amount of energy needed to extract the most energetic electron from the tip to the vacuum.

\[ \rightarrow \text{tunneling prob. for closest atom} \approx 400 \]