Bryn Mawr Physics 302- Advanced Quantum Mechanics - 2010
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Assignment #4 (Corrected)
Due: Friday, Feb. 19 at 10:00 am.
Reading: Chapter 4 of Townsend

Note: As Townsend points out on p. 80, operators such as $\hat{J}_z$ can refer to angular momentum due to spin or orbital angular momentum. When specifically referring to spin, they are called, for example, $\hat{S}_z$. So, for this assignment you can think of $\hat{S}_z$ as being equivalent to $\hat{J}_z$.

Group problems:
3.14 (You may wish to wait until after class Tuesday to do this. Use raising and lowering operators.)
3.15 Hint: For example, you can find $|+x\rangle$ in the z-basis using $\hat{S}_x |+x\rangle = \hbar |+x\rangle$. Don’t forget to normalize your results, e.g. set $\langle +x | +x \rangle = 1$.
3.17 Do problem 3.15 before doing part c of this problem.
4A. In class, we used an arm-waving argument (literally) to show the reasonableness of $[\hat{J}_x, \hat{J}_y] = i\hbar \hat{J}_z$. In this problem, you will quantify our argument.
   i) Explain briefly why the mathematical version of what we showed in class would be $\hat{R}(\alpha \hbar) \hat{R}(d\varphi \hat{j}) \hat{R}(d\varphi \hat{i}) = \hat{R}(d\varphi \hat{i}) \hat{R}(d\varphi \hat{j})$, where $\alpha$ is to be determined and $d\varphi$ is an infinitesimal rotation. (In class, we used $d\varphi = 45^\circ$, which is hardly infinitesimal, but similar results would apply even better to smaller rotations.) Note: in writing this, we have ignored a small difference in the vertical displacement between the left and right hands; we’ll deal with this in part iii below.
   ii) Using diagrams and equations, show that $\alpha = (d\varphi)^2$.
   iii) Using diagrams and equations, show that the vertical displacement ignored in part i is of order $(d\varphi)^3$ or higher.
   iv) Now show that the relation in part i leads quantitatively to $[\hat{J}_x, \hat{J}_y] = i\hbar \hat{J}_z$. Don’t forget that $d\varphi$ is very small.

4B. i) In class, I introduced corrected kets for spin-$\frac{1}{2}$ particles, ones that are consistent with cyclic permutations. Using these kets, confirm that $\hat{R}\left(\frac{\pi}{2} \hat{j}\right) |+z\rangle = e^{i\delta} |+x\rangle$, where $e^{i\delta}$ is an (unimportant) overall phase factor, as would be expected from considering how the rotation affects ordinary vectors.
   ii) Use the corrected kets to show that $\hat{R}\left(\frac{\pi}{2} \hat{j}\right) \hat{R}\left(\frac{\pi}{2} \hat{i}\right) |+z\rangle = |-z\rangle$, as would be expected.

Individual-problems: 2.11, 3.10