Physics 308: Advanced Classical Mechanics  
Bryn Mawr College, Fall 2011  
Problem Set 2

Distributed:  Thursday, September 8, 2011.  
Due:  Thursday, September 15, 2011 by the start of class.

Reading

For Tuesday, please read Sections 4.1 – 4.4 in Taylor. For Thursday, read Sections 4.6 – 4.10.

This problem set has more “meaty” problems than the first problem set, although there are fewer of them. In each case draw a picture of the scenario. And then spend time thinking (not writing) about which fundamental principles are at play (e.g. conservation of energy or momentum or both and/or others?). As always, I encourage you to work collaboratively on these problems (with the exception of the problem labeled “INDIVIDUAL PROBLEM,” which must be completed by you alone).

Problems

1. **Propelling a car:** You decide to throw baseballs at a car of mass $M$ that is free to move without friction on the ground. You throw the balls at the back of the car at speed $u$. The balls bounce elastically directly backward off the back window. Let the instantaneous velocity of the car be $v$.

   (a) Assuming that the car is much more massive than the baseball, what is the change in the baseball’s momentum from a single collision?

   (b) Now assume that the baseballs leave your hand at a mass rate of $\sigma$ kg/s (invoking a continuous rate rather than thinking about discrete baseball impacts actually simplifies the problem). What is the mass rate of incident baseballs seen by the car?

   (c) If the car starts at rest, find its speed $v$ as a function of time.

2. **Breaking Up:** A ball is fired from level ground and lands in a hole 100 meters away. A second ball is launched in exactly the same way. But this time, in mid-flight, the ball unexpectedly breaks into two equal pieces. One piece lands 100 meters beyond the hole.

   (a) If the pieces land at the same time, where does the second piece land?

   (b) How does the situation change if the pieces land at different times? Consider both cases (the piece 100 m beyond the hole landing earlier and later than the other piece). Compare the landing positions in this question to the answer you derived in part (a).

3. **INDIVIDUAL PROBLEM – Eagle Balancing Toy:** A judicious choice of mass distribution allows the eagle toy pictured below to balance by its beak.
The drawing on the right is a simplified model of this toy: a uniform rod of mass $M$, shaped into a semi-circle of radius $L$ (the body) with two point-masses, $m$, attached to the ends of the rod (the wing tips). Using the center of the circle as your origin, compute the center of mass of the model $\vec{R}_{CM} = (X_{CM}, Y_{CM})$. What is $Y_{CM}$ if $m = M$?

4. **Moment of Inertia of a Uniform Disk**: Calculate the moment of inertia of a uniform disk of mass $M$ and radius $R$ rotating about an axis that is normal to the disk through its center of mass. Call this moment of inertia $I_{CM}$. Now, calculate the moment of inertia for the disk if it rotates about an axis that is still normal to the disk, but shifted to be tangent to the edge of the disk. Call this $I_P$. A very useful theorem called the parallel-axis theorem states: $I_P = I_{CM} + MR^2$. This means that once you’ve calculated the moment of inertia for an object around an axis that passes through the body’s center of mass ($CM$), then to calculate the moment of inertia around a parallel axis, you simply have to add $MR^2$, where $M$ is the mass of the body and $R$ is the distance between the two (parallel) axes. **Warning**: this theorem is only valid with the center of mass, and not with any other point. However, this theorem holds for bodies of arbitrary shape.

5. **Rolling, rolling, rolling**: A disk is placed on a plane that is inclined by an angle $\theta$ above the flat ground. The disk is released from rest and begins rolling without slipping down the plane.

   (a) Draw this system and the forces at play.

   (b) What force causes the disk to roll? Is there a unique answer? (**Hint**: Consider both the center of mass of the disk and the point of contact of the disk).

   (c) What is the linear acceleration $\ddot{v}$ of the disk?

You will find the results from Problem 4 useful.

END