Physics 308: Advanced Classical Mechanics
Bryn Mawr College, Fall 2011
Problem Set 4

Distributed: Friday, September 23, 2011.
Due: Friday, September 30, 2011 by noon.

Reading

For Tuesday and Thursday, please finish Chapter 5 in Taylor. Although this is the last homework set before fall break, please do read Chapter 6 about the Calculus of Variations for October 4th and 6th.

As always, I encourage you to spend time thinking (not writing) about which fundamental principles are at play (e.g. conservation of energy or momentum or both and/or others?). As always, I encourage you to work collaboratively on these problems (with the exception of the problem labeled “INDIVIDUAL PROBLEM,” which must be completed by you alone).

You’ll also notice a few “optional” problems. I will not grade your responses to these problems, but it is highly recommended that you work through them as they are very relevant to this course.

Problems

1. Potential Energy of Atoms in a Molecule: Do Taylor Problem 5.2 which studies the Morse function as a model for the potential energy between atoms in a molecule:

\[ U(r) = A \left( \left( e^{(R-r)/S} - 1 \right)^2 - 1 \right) \]  

2. Mass on a Spring: Do Taylor 5.8. You will compute the natural oscillation frequency of the system, and solve for the motions of the particle \( x(t) \), given a set of initial conditions \( x(0) \) and \( \dot{x}(t) \).

3. The Period and Decay Envelope of a Damped Oscillator: Taylor Problem 5.25.

4. Critically Damped and Over-damped Systems Do Not Oscillate: Do Taylor Problem 5.27 part (a) to show that a critically damped oscillator can never pass through the origin \( x(t) = 0 \) more than once. You are not required to do part (b) of the problem. But do so if you wish to show that over-damped systems also do not oscillate.

5. Damped & Driven Oscillator: Do Taylor 5.37 which shows that although the short-term behaviour of a damped and driven system may be complicated, the long-term behaviour is dictated by the driving force. Please use Mathematica to plot your results.

6. (INDIVIDUAL) A Car is a Damped & Driven Oscillator: Do Taylor 5.43 to show that you can estimate the natural frequency of a car using a few empirical measurements.
7. (OPTIONAL) Various Ways to Represent Simple Harmonic Motion: Do Taylor 5.5 to show that the following forms of simple harmonic motion are equivalent, and find the relationship between the constants.

\[ x(t) = C_1 e^{i\omega t} + C_2 e^{-i\omega t} \]
\[ = B_1 \cos(\omega t) + B_2 \sin(\omega t) \]
\[ = A \cos(\omega t - \delta) \]
\[ = \text{Re} \{ C e^{i\omega t} \} \]

8. (OPTIONAL) Complex Numbers Math Review: Do Taylor 5.35 to refresh your memory about complex numbers.

9. (OPTIONAL) Resonance: Do Taylor Problems 5.40 and 5.41 to study a system that is driven at or near its resonant frequency.

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