Due: Fri 27 Jan 2017

1. Radiation from two hemispheres at opposite potential. Jackson Problem 9.3.


Unlike most of Chapter 9, this problem does not assume harmonic time dependence. Note that there is a typo in part (b): The quoted $B(x,t)$ has an extra factor of $1/c$.

4. Useful results involving $L$. Please verify the following three identities which come up in Chapter 9:

$$\nabla = \hat{r} \frac{\partial}{\partial r} - \frac{i}{r^2}(r \times L), \quad (1)$$

$$\nabla(r \cdot \nabla) = \nabla + \left(r \frac{\partial}{\partial r}\right) \nabla, \quad (2)$$

$$i\nabla \times L = r\nabla^2 - \nabla \left(1 + r \frac{\partial}{\partial r}\right). \quad (3)$$

**General comments:** Problems 1, 2, 3 are all to be worked out the electric dipole approximation. However, in the third problem, you are additionally asked to prove that the magnetic contribution vanishes to all multipole orders. (See below.)

**Comments on Jackson Problem 9.10:** Part (a) of Problem 9.10 alludes to later material in Sec. 9.10, in which the notation is slightly different than in Secs. 9.1–9.3.

In Sec. 9.3, Jackson defines the magnetization to be

$$\mathcal{M} = \frac{1}{2} r \times J. \quad (9.32)$$

Then, later in Sec. 9.10, he changes notation. He reserves the symbol $\mathcal{M}(x,t)$ for the intrinsic magnetization (for example, due to intrinsic spin), and writes the magnetization due to $J$...
explicitly as \( \frac{1}{2} \mathbf{r} \times \mathbf{J} \) everywhere it appears. This is the reason for the funny notation \( \mathcal{M}(\mathbf{x}, t) \) with quotes, in Problem 9.10:

\[
\mathcal{M}(\mathbf{x}, t) = \frac{1}{2} \mathbf{r} \times \mathbf{J}.
\]

Since Jackson wants \( \mathcal{M}(\mathbf{x}, t) \) to denote the intrinsic magnetization, he needs a new symbol \( \mathcal{M}''(\mathbf{x}, t) \) for \( \frac{1}{2} \mathbf{r} \times \mathbf{J} \). You could call this the “orbital” magnetization.

In Problem 9.10, we neglect the intrinsic spin and intrinsic magnetization. Part (a) of the problem employs Eq. (9.172) of Sec. 9.10 in the following minimal way, that does not rely on technology of Secs. 9.7–9.10. You are asked to compute \( \nabla \cdot \mathcal{M}'' = \frac{1}{2} \nabla \cdot (\mathbf{r} \times \mathbf{J}) \), which you should find vanishes. Then, without reading any of Sec. 9.10, you can take a look at Eq. (9.172) and see what this implies for the magnetic multipole moments \( M_{lm} \).