Due: Wed 4 Feb 2009.

Reading:

Fri 1/30: Please review electromagnetism, and div, grad, curl, as necessary.

Mon 2/2: Townsend Secs. 1.4.

Wed 2/4: Townsend Secs. 1.5.

Problems (continued pages 2-4):

1. **Relativistic conservation laws.** Townsend Problem A.20 (p.492).

2. **Doppler shift in the classical (Thomson) scattering of light by an electron.**

   For a plane wave of frequency $\nu_{\text{wave}}$ and speed of propagation $c$ (see Fig. 1(a) below), the classical Doppler effect says the frequency detected by an observer moving away from the wave at speed $v_{\text{obs}}$ is

   $$\nu_{\text{obs}} = (1 - \frac{v_{\text{obs}}}{c})\nu_{\text{wave}}.$$  \hspace{1cm} (1)

   A moving point source emits a spherical wave rather than a plane wave (see Fig. 1(b)). In this case, the classical Doppler effect says that the frequency detected by an stationary observer at an angle $\theta$ relative to the direction of motion of the source is

   $$\nu_{\text{obs}} = \frac{1}{1 - \frac{v_{\text{source}}}{c}\cos \theta}\nu_{\text{source}},$$ \hspace{1cm} (2)

   where $\nu_{\text{source}}$ is the frequency emitted according to the source, and $v_{\text{source}}$ is the speed of the source.

   In classical Thomson scattering, an electron moving at speed $v$ absorbs light from an incident plane wave (as in Fig. 1(a)). It emits light in a spherical wave at the same frequency according
to the electron. (a) By combining the Doppler effect formulae given above, show that the wavelength $\lambda'$ of the scattered light detected by a stationary observer at angle $\theta$ is

$$\lambda' = \frac{1 - \frac{v}{c} \cos \theta}{1 - \frac{v}{c}} \lambda, \quad (3)$$

where $\lambda$ is the wavelength of the incident light. (b) Working to first order in $v/c$, derive the classical result quoted in class:

$$\lambda' - \lambda \simeq \frac{v}{c} (1 - \cos \theta). \quad (4)$$

You will need to use the approximation $(1 - x)^{-1} \simeq 1 + x$ for small $x$.

3. Wave equation for $\vec{E}$ and $\vec{B}$ from Maxwell’s equations. Maxwell’s equations are

$$\nabla \cdot \vec{D} = 4\pi \rho, \quad \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t},$$

$$\nabla \times \vec{H} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}, \quad \nabla \cdot \vec{B} = 0. \quad (5)$$

(a) In the absence of sources ($\rho, \vec{J} = 0$), express these equations in terms of $\vec{E}$ and $\vec{B}$ only.

(b) To obtain the wave equation for $\vec{E}$, first solve one of the Maxwell equations for $\partial \vec{E}/\partial t$, and then differentiate both sides with respect to $t$ to obtain an expression for $\partial^2 \vec{E}/\partial t^2$. 

Figure 1: (a) Absorption of light and (b) emission of light by a moving electron.
(c) In the result of (b), note that spatial and time derivatives commute (i.e., ordering is unimportant):
\[
\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{V}) = \vec{\nabla} \times \frac{\partial \vec{V}}{\partial t},
\]
for any vector \( \vec{V} \). Apply this observation to eliminate \( \vec{B} \) from the result of part (b), using the Maxwell equation for \( \partial \vec{B}/\partial t \).

(d) Finally, for the triple vector product, we have the identity
\[
\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} (\vec{a} \cdot \vec{c}) - (\vec{a} \cdot \vec{b}) \vec{c},
\]
and similarly
\[
\vec{\nabla} \times (\vec{\nabla} \times \vec{V}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{V}) - \nabla^2 \vec{V}.
\]
Use this identity, together with one of the remaining two Maxwell equations, to obtain the wave equation for \( \vec{E} \):
\[
\nabla^2 \vec{E} - \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0,
\]
where \( v = c/n \) and \( n = 1/\sqrt{\varepsilon \mu} \).

(e) By an analogous series of steps, derive the wave equation for \( \vec{B} \):
\[
\nabla^2 \vec{B} - \frac{1}{v^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0.
\]

4. Poynting vector for a plane wave. For an electromagnetic plane wave propagating in the \( \hat{z} \) direction with \( \vec{E} \) plane-polarized in the \( \hat{x} \) direction, the fields are
\[
\vec{E} = E_0 \hat{x} \cos(kz - \omega t) \quad \vec{B} = B_0 \hat{y} \cos(kz - \omega t).
\]

(a) Show that the Poynting vector is
\[
\vec{S} = \frac{v\varepsilon}{4\pi} E_0^2 \hat{z} \cos^2(kz - \omega t).
\]

(b) What is the time average of \( \vec{S} \)?

5. Double slit diffraction. In a double slit diffraction experiment, yellow helium light of wavelength \( \lambda = 587.6 \) nm produces fringes of separation \( \Delta y = 0.50 \) mm on a screen 2.25 m away. What is the distance between the slits? (Hint: you will need to use the small angle approximation \( \sin \theta \approx \tan \theta = y/L \).)
6. **Single slit diffraction.** Townsend Problem 1.3.

7. **Feedback.** By Wednesday of each week, please send me an email message to provide feedback on the class and on your reading. (My email address is mbschulz at brynmawr.edu). For example: Which parts were easier or harder to understand? Do you have any questions that you would like to clarify or areas where you would like more practice in recitation section? Was there something that you found particularly interesting or uninteresting? As you know, we are using a prepublication version of the textbook. If you have any thoughts on how to improve the textbook for future students taking this class, please let me know and I will pass that information on to the author, John Townsend.