Problem 2.3

Our discussion of the He-atom interferometer focused on the location of the interference fringes. In addition, the interference pattern is modulated by a single slit diffraction pattern. Determine the width of this diffraction envelope, that is, calculate the distance in the detecting plane between the first nodes of the diffraction envelope on either side of the central maximum. How many interference fringes fit in this envelope for a wavelength of 45 pm? Compare your results with the data in Fig 2.4

(Data from chapter 2: double slit separation a = 8 μm,
width of each slit b = 1 μm,
distance to screen L = 1.95 m)

Solution

Double slit maxima: a sin θ = nλ  (a = separation)
Single slit minima: b sin θ = mλ, m ≠ 0  (b = slit size)

1st single slit min: b sin θ = ±λ

sin θ = ±λ/b

Order of double slit max of this angle: n = a sin θ/λ = ± a/b

a = 8 μm, b = 1 μm => n = ±8

So, 15 double slit maxima should be visible  (n = -7 to n = +7)
The double slit maxima at n = ±8 are suppressed since they are located at nodes (minima) of the single slit envelope.

[Diagram showing single slit envelope, n=2 double slit max, m=1 single slit node, n=8 suppressed double slit max, 15 double slit maxima]
Distance between nodes on detecting plane Z.

\[ \frac{\sin \theta}{\tan \theta} = \frac{Y}{L} \]

So, nodes of single slit envelope at

\[ b \sin \theta = m \lambda \Rightarrow b \frac{Y}{\lambda} = m \]

\[ Y_m = m \frac{2L}{b} \]

\[ Y_{m+1} - Y_{m-1} = \frac{2L}{b} \]

\[ = \frac{2}{b} \left( 45 \times 10^{-9} m \sqrt{1.95 m} \right) = 1.8 \times 10^{-4} m = 0.18 \text{mm} \]

\[ = 180 \mu m \]

Fig 2.4.² See 6 peaks left of center, 7 right of center

\[ \Rightarrow \text{rough agreement between our results on figure} \]

Would be nice to see more detail to left and right in Fig 2.4
to see nodes of envelope more clearly.

Edge effects? Node at \( n = 6 \)² or \( n = 7 \)²

Note how small the wavelength is: \( \lambda = 45 \text{pm} \), compared
to hundreds of nm for visible light. Similarly, the width
of the single slit envelope 180 \( \mu m \) is also very small compared
to the width in analogous experiments with light.

Very small wavelengths are typical for matter waves.

That's why it took longer to notice the wave properties of matter.