Physics 214: Introduction to Quantum Mechanics
Spring 2013

Problem Set 3

Due: Wed 13 Feb 2013

Reading:

Fri 2/8: Townsend Secs. 2.1 and 2.2.

Mon 2/11: Townsend Sec. 2.3.

Wed 2/13: Townsend Sec. 2.7.

Problems:

1. Double slit diffraction with slits of different width. Townsend Problem 1.29.

2. Three slit diffraction grating. Townsend 1.34.


4. Diffraction from two slits of finite width.

(a) Townsend Problem 2.3. (See p. 52 for the width $b$ of each slit and their separation $a$. To relate the angle $\theta$ to the position $x$ on the detection plane, use the small angle approximation $\sin \theta \approx \tan \theta = x/L$, as on p. 54.)

(b) Please write an expression (valid for small $\theta$) for the probability distribution $P(x)$ of observing a photon at position $x$ in the experiment of part (a). Since the wavelength, slit ratio $a/b$, and screen distance $L$ are known, you can express the ratio $P(x)/P(0)$ as an explicit function of the rescaled position $\Phi = kb\theta/L$. Using Mathematica, please plot this function of $\Phi$ together with its envelope on the same set of axes. A good range of $\Phi$ to use is $-6\pi \leq \Phi \leq 6\pi$. The plot should look similar to the figure that we drew in class today. If the plot comes out too small, you can click and drag it to make it larger.
To expand your knowledge of *Mathematica*, the Help menus is an excellent resource. You should get into the habit of using it, and may be asked to do so on future problem sets! For now, here is an example that illustrates what you need to know for this problem. (In the problem, the functions that you actually want to plot are different from those in the example.) Here's the example: the *Mathematica* code that follows shows how to define two functions f[] and g[] and then then plot them on the same set of axes using Plot[]. Note that built-in functions start with an uppercase letter.

```
f[u_] := Cos[u]
g[u_] := Sinc[u]
Plot[{f[u], g[u]}, {u, -6 Pi, 6 Pi}, PlotStyle->{Dashed, Thick}, PlotRange->All]
```

5. **Transmission interference from a thin glass plate.** Consider a thin glass plate of thickness \(d\) and index of refraction \(n\) surrounded by vacuum above and below. A photon of wavelength \(\lambda\) is normally incident on the top surface from a distance \(d_1\) above the plate. A detector is located a distance \(d_2\) below the bottom surface. The transition amplitude for transmission at either interface is \(t\), and for reflection is \(\pm r\), where the sign depends on whether the reflection is hard or soft. In recitation section we computed (but did not simplify) the total probably amplitude \(z\) to observed a reflected photon. In this problem we instead consider the transmitted photon.

(a) What is the probability amplitude \(z_t\) to observe a transmitted photon at the detector? Include the contributions from all possible paths, taking into account multiple reflections.

(b) The transition amplitudes \(r\) and \(t\) are not independent. Please explain why, and give an equation relating \(r\) and \(t\).

(c) What is the probability \(P\) corresponding to the probability amplitude found in part (a)? Using part (b), show that the result can be expressed in the form

\[
P = \frac{1}{1 + \frac{4|r|^2}{(1 - |r|^2)^2} \sin^2(k'd)}
\]

where \(k' = 2\pi/\lambda' = 2\pi n/\lambda\) is the wavenumber in the glass. (Depending on your method, the relation \(1 - \cos 2x = 2\sin^2 x\) might come in handy.)

This problem has a practical application: the same analysis applies to the operation of the Fabry-Pérot interferometer, an instrument that you might use during your time at Bryn
Mawr. For a Fabry-Pérot, the only difference is that the reflections occur between two partially reflecting surfaces separated by empty space (rather than glass). If \(|r|\) is taken very close to 1, then the maxima of Eq. (1) are very narrowly peaked. This makes the Fabry-Pérot interferometer a good instrument for precise measurements of wavelength.

6. Integral derivation of single slit diffraction

In class, we derived the probability amplitude for single slit diffraction from that of a diffraction grating of \(N\) slits. In the limit \(N \to \infty\), a grating of fixed total width \(a = (N - 1)d\) becomes a continuous slit of width \(a\). In this problem, we compute the single slit result directly from a sum over all possible paths.

(a) Consider a point \(P\) on the screen at an angle \(\theta\) from the horizontal. Let \(x\) be a coordinate along the slit that runs from 0 at the top of the slit to \(a\) at the bottom of the slit. Explain why the probability amplitude for a path starting a position \(x\) and ending at \(P\) is

\[
z(x, \theta) = re^{ik(d_1 + x \sin \theta)},
\]

where \(d_1\) is the distance from the top of the slit to \(P\).

(b) The total probability amplitude for observation of a photon at \(P\) is obtained by summing \(z(x, \theta)\) over all possible starting points \(0 \leq x \leq a\). The coefficient \(r\) in equation (2) must be infinitesimal, that is, \(r = C\,dx\) for some constant \(C\), in order to obtain a finite result. Then,

\[
z_P = \int_0^a e^{ik(d_1 + x \sin \theta)} C\,dx = Ce^{ikd_1} \int_0^a e^{ik \sin \theta} x \, dx
\]

Please evaluate this integral. Show that the result is

\[
z_P(\theta) = z_P(0)\, e^{i\Phi/2} \sin \frac{\Phi}{2},
\]

where \(\sin u = (\sin u)/u,\, z_P(0) = Ca e^{ikd_1},\) and \(\Phi = ka \sin \theta\).

(c) What is the corresponding probability?

7. Feedback. By Thursday of each week, please send me an email message to provide feedback on the class and on your reading. (My email address is mbschulz at brynmawr.edu). For example: Which parts were easier or harder to understand? Do you have any questions that you would like to clarify or areas where you would like more practice in recitation section? Was there something that you found particularly interesting or uninteresting? Was the problem set of reasonable length and difficulty. If you have any thoughts on how to
improve the textbook for future students using future editions, please let me know and I will pass that information on to the author, John Townsend. The purpose of the feedback is to help you reflect on your learning process and to provide me with brief but valuable information that will help to make this class the best possible experience for everyone.