Comments on Townsend Problem 4.9 (PS.8 Problem 5)

Semi-infinite potential:
Same for $x > 0$

Finite square well:
odd $\psi(x)$ satisfy $\psi(0) = 0$

Compared to a finite square well $V(x) = \begin{cases} -V_0 & -a < x < 0 \\ 0 & \text{otherwise} \end{cases}$, the given potential has

- The same $V(x)$ for $x > 0$
- The same condition $\psi(x) > 0$ as $x \to \infty$ for bound states
- But, require $\psi(0) = 0$ since $V(x) = \infty$ for $x \leq 0$
Comments on Townsend Problem 4.9 (continued)

Recall the two cases for the finite square well:

Case I: \( A = B, \ C = D, \ \cos k x \) in region II, even \( \psi(x) \)
\[ => \text{don't satisfy } \psi(0) = 0. \]

Case II: \( A = -B, \ C = -D, \ \sin k x \) in region II, odd \( \psi(x) \)
\[ => \text{do satisfy } \psi(0) = 0. \]

General rule: Take any even potential, replace \( x \leq 0 \) potential
with \( V(x) = \infty \). Odd \( \psi_n(x) \) are energy eigenfunctions of the
new problem, for \( x \leq 0 \).

Second comment: For a piecewise constant potential,
\[ k_i = \sqrt{\frac{2m(E-V_i)}{\hbar^2}} \text{ in classically allowed regions } E > V_i, \]
\[ k_i = \sqrt{\frac{2m(V_i-E)}{\hbar^2}} \text{ in classically forbidden regions } E < V_i. \]

So,
\[ k = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} \text{ in region II, since } E > V_\text{II} = -V_0, \]
\[ k = \sqrt{\frac{2m(-E)}{\hbar^2}} \text{ in region III, since } E < V_\text{III} = 0. \]

Treat the finite square well problem (for a well of width 2a, \(-a < x < a\))
then just keep odd wavefunctions
\[ \xi = \frac{k(2a)}{2} = \frac{2a}{2} \sqrt{\frac{2m(E+V_0)}{\hbar^2}}, \]
\[ \xi_0 = \frac{2a}{2} \sqrt{\frac{2mV_0}{\hbar^2}}. \]
\[ => \quad \frac{k(2a)}{2} = \sqrt{\xi_0^2 - \xi^2}. \]
Comments on Townsend Problem 4.9 (continued)

All E's are shifted by $-V_0$ compared to class March 17-19, but matching conditions give same equation in $\xi$ as in odd case in class on March 19.

(b) See notes 3/19/10.

How big does $\delta_0$ have to be for at least one solution to exist? This gives an inequality $\delta_0 \geq$ something. Use definition of $\delta_0$. $\Rightarrow (\ldots) V_0 \geq$ Something.

(c) Use the result of part (b).