Problem

Light of wavelength 6500 Å falls from narrow slit onto two parallel slits whose separation is 0.150 mm. (a) What color is the light? (b) How far apart are the interference maxima at a distance of 30.0 cm?

(a) $\lambda = 10^{-10} \text{ m} = 10^{-1} \text{ Å}$, so $\lambda = 6500 \text{ Å} = 650 \text{ nm}$

Visible region: 400 - 700 nm $\Rightarrow \lambda$ is real. (See also fig. 1.3)

(b) Double slit maxima:

path difference $d_2 - d_1 = a \sin \theta = n \lambda$

For small $\theta$, $\sin \theta \approx \tan \theta = \frac{y}{L}$

$\Rightarrow a \frac{y}{L} = n \lambda$

$y_0 = n \frac{aL}{\lambda}$

$\Delta y = \frac{2L}{\lambda}$

$\Delta y = \frac{(6.50 \times 10^{-7} \text{ m})(0.80 \text{ m})}{(1.50 \times 10^{-4} \text{ m})} = 3.5 \times 10^{-3} \text{ m} = 3.5 \text{ mm}$

Problem

Eight interference fringes on a screen 100 cm away from an illuminated double slit of 0.280 mm separation have a separation width of 1.95 cm. Find the wavelength. What color is the light?

Solution From the previous problem, $y_1 = \frac{2L}{\lambda}$

$y_2 - y_1 = \pm \frac{2L}{\lambda}$

$\lambda = \frac{y_2 - y_1}{2L} = \frac{(2.00 \times 10^{-4} \text{ m})(1.95 \times 10^{-2} \text{ m})}{7(1.00 \text{ m})} = 5.57 \times 10^{-7} \text{ m}$

$= 557 \text{ nm yellow}$
Problem 1.18

(a) Express the complex number $z_1 = (\sqrt{3}+i)/2$ in the form $r e^{i\phi}$. What about $z_2 = (1+i\sqrt{3})/2$?

(b) If these complex numbers are the probability amplitude for a photon to be detected, what is the probability in each case?

Solution

$z = x + iy = r e^{i\theta}$

- $z_1 = \frac{\sqrt{3}}{2} + \frac{i}{2}$
  \[ r = \sqrt{x^2 + y^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} \]
  \[ \cos \phi = \frac{x}{r} = \frac{\sqrt{3}}{2} \]
  \[ \sin \phi = \frac{y}{r} = \frac{1}{2} \]
  \[ r = 1 \]
  \[ \phi = \frac{\pi}{6} \text{ (i.e., 30°)} \]

$\Rightarrow z_1 = e^{i\pi/6}$

- $z_2 = \frac{1}{2} + \frac{i\sqrt{3}}{2}$
  \[ r = \sqrt{x^2 + y^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} \]
  \[ \cos \phi = \frac{x}{r} = \frac{1}{2} \]
  \[ \sin \phi = \frac{y}{r} = \frac{\sqrt{3}}{2} \]
  \[ \phi = \frac{\pi}{3} \text{ (i.e., 60°)} \]

$\Rightarrow z_2 = e^{i\pi/3}$

(b) Probability $P = z^* z$ is 1 in either case:

$P_1 = e^{-i\pi/6} e^{i\pi/6} = 1$
$P_2 = e^{-i\pi/3} e^{i\pi/3} = 1$. 