Problem 1.23

One photodetector is located in front of a thick piece of glass and another photodetector is located within the glass. The glass reflects 4% of the light. (a) What is the magnitude of the prob. amplitude for reflection of the photon? (b) for transmission?

Solution

\[ P_r + P_t = 1 \]
\[ P_r = 0.04 \quad (4\%) \]
\[ (\text{prob. reflected}) \quad (\text{prob transmitted}) \Rightarrow P_t = 0.96 \quad (96\%) \]

(a) \[ P_r = |\mathbf{z}_r\rangle \langle \mathbf{z}_r| = |\mathbf{z}_r|^2 = 0.04 \]
\[ \Rightarrow |\mathbf{z}_r| = 0.2 \]

(b) \[ P_t = |\mathbf{z}_t\rangle \langle \mathbf{z}_t| = |\mathbf{z}_t|^2 = 0.96 \]
\[ \Rightarrow |\mathbf{z}_t| = 0.98 \]

If asked about phase:

Air \to \text{glass} = \text{hard reflection} \quad (\text{glass} \to \text{Air})
\[ \Rightarrow \text{relative phase of } \pi \text{ between reflected and transmitted amplitudes.} \]

Problem 1.32

Add the two complex numbers \( \mathbf{z}_1 = 1 \) and \( \mathbf{z}_2 = e^{i\pi/3} \) by:
(a) adding the real and imaginary pieces together and (b) using geometry to "add the arrows" representing each of these complex numbers.
Solution

(a) \[ z_1 = 1 \]
\[ z_2 = e^{i\pi/3} = \cos \frac{\pi}{3} + i\sin \frac{\pi}{3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}. \]

Sum: \[ z = z_1 + z_2 = \frac{3}{2} + i\frac{\sqrt{3}}{2}. \]

\[ (z = x + iy) \]

To compare with part (b), need magnitude and phase: \[ z = re^{i\phi}. \]
\[ r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\sqrt{3}^2}{2} = \frac{3}{2} \Rightarrow \phi = \pi/6 \text{ (or 30°)}. \]

(b) \[ |z_1| = |z_2| = 1 \]
\[ \phi_1 = 0, \quad \phi_2 = \frac{\pi}{3} \text{ (or 60°)}. \]

"Vector sum":
\[ r = z_1 + z_2 = x + iy \]

\[ 120^\circ \quad \text{60°} \]

\[ \Rightarrow r = 2 \cos 30^\circ = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3} \]
\[ \phi = 30^\circ = \frac{\pi}{6}. \]