Problem Set 3 Hints and Clarifications


Please interpret the word “maxima” as the principal maxima of the diffraction grating. These are the maxima in which our probability formula in class gave zero over zero and we had to use L’Hopital’s rule.

The principal maxima have a simple interpretation: The path length difference between neighboring paths is an integer multiple of the wavelength.

So, for this problem, you do not actually have to compute the probability amplitude and find it’s maxima and minima. You can just use the condition in the previous paragraph.

4. Diffraction from two slits of finite width. Part (b):

In class, we found that the probability of observing a photon at angle theta for double slit diffraction (which I’ll abbreviate as ‘ds’) is

$$P_{ds}(\theta) = 4r^2 \cos^2(\delta/2),$$

where $\delta = ka \sin \theta$. Since $P_{ds}(0) = 4r^2$, we can also write this as

$$P_{ds}(\theta) = P_{ds}(0) \cos^2(\delta/2).$$

Similarly, for single slit diffraction (which I’ll abbreviate as ‘ss’) we found

$$P_{ss}(\theta) = P_{ss}(0) \text{sinc}^2(\delta_{ss}/2),$$

where $\delta_{ss} = kb \sin \theta$.

For diffraction from two slits of finite width, we said the interference pattern looks like the $\cos^2$ of double slit diffraction, inside a $\text{sinc}^2$ single slit envelope. In terms of equations, the probability is just the product of the two expressions above:

$$P(\theta) = P(0) \cos^2(\delta/2) \text{sinc}^2(\delta_{ss}/2).$$
We used $y$ for the location on the screen, but Townsend uses $x$, so the problem uses his notation. Use the small angle approximation $\sin \theta \simeq x/L$ in your expressions for $\delta$ and $\delta_{ss}$. It is convenient to express both in terms of the quantity $\Phi$ defined in the problem, and the ratio $a/b$. Then, from the data in the book for $a$ and $b$, the quantity $a/b$ is known, and you know the function $P(\theta)/P(0)$ explicitly. You can write it down as an explicit function of $\Phi$, and plot it using Mathematica.

5. **Transmission interference from a thin glass plate.**

The probability amplitude is the sum over paths as usual. However, there is an infinite number of paths. The photon can go straight through. It can reflect back and forth once inside the glass plate and then go through. It can reflect back and forth $n$ times before transmission, for $n = 0, 1, 2, \ldots, \infty$. You should see a pattern. The amplitude for the path with $n + 1$ pairs of reflections is always obtained from that for $n$ pairs of reflections, by multiplying by the same factor. So, the sum over paths is an infinite geometric series, which you know how to sum.

You will obtain some fraction $z = A/B$ for the total probability amplitude and can then compute

$$z^*z = \frac{A^*A}{B^*B}$$

To obtain the desired expression, you’ll need to do some manipulation, including the following: Use part (b) to re-express $t^2$ in terms of $r^2$, and use the trigonometry identity that relates $\cos(2k'd)$ to $\sin^2(k'd)$.

Since we are assuming $t$ and $r$ are real in this problem, there is no distinction between $t^2$ and $|t|^2$, or between $r^2$ and $|r|^2$. 